SYLLABUS COVERED

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.

INDEX

PART – I FORCES, FIELDS AND DIPOLE

PART – I FORCES, FIELDS AND DIPOLE

I.1 ELECTRIC CHARGE

Electric charge is a property of matter that causes it to produce and experience electrical and magnetic effects.

Experiments on frictional electricity allows us to infer that there are two types of charges, positive and negative. Like charges repel and unlike charges attract.

By convention, the charge on glass rod rubbed with silk is assigned positive value, then the charge on a plastic rod rubbed with fur is negative.

Old names are vitreous (glass like, positive) and resinous (resin like, negative).

When electrons are removed from a neutral body, the body develops positive charge.

When electrons are captured by a neutral body it acquires negative charge.

Since electrons possess mass, charging a body (removal or addition of electrons) in principle, implies change in mass of the body. When electrons are added mass increases in principle.

A neutral object possesses equal number of positive and negative charges.

The **SI unit of charge** is coulomb (C).

One **coulomb** is that amount of charge which flows through any cross section of a conducting wire in one second when the electric current flowing through the wire is one ampere.

The electric charge on an electron (up to two significant figures) is $q_{electron} = -1.6\times10^{-19}$ C .

The electric charge on a proton (up to two significant figures) is $q_{proton} = +1.6\times10^{-19}$ C .

The coulomb is a big unit. Other units are

micro coulomb $1\,\mu C = 10^{-6} C$,

nano coulomb $1\,n\,C = 10^{-9} C$,

pico coulomb $1\,pC = 10^{-12} C$.

The **dimensions** of the charge are $M^0L^0T^1A^1$ or $M^0L^0T^1I^1$.

The electric charge is a scalar quantity.

Comment:

A gold leaf electroscope (Fig.1.1) is a device that detects charge and its sign.

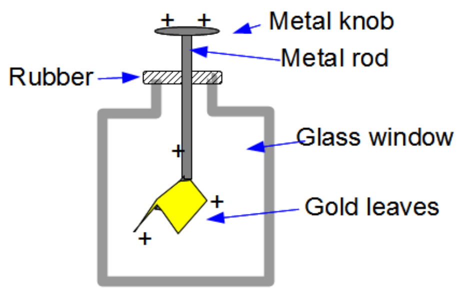


Fig.1.1 Gold leaf electroscope

Conductor and Insulator

Conductors allow movement of electric

charges through them, insulators do not.

A **conductor** is a material that readily allows flow of electric charge through it.

Inside a conductor there are a large number of electric charges that are relatively free to move and conduct electricity.

Examples of conductors: Metal, human body, animal body, earth, electrolytic solution, ionized gas.

In metals, mobile charges are electrons. About one electron per atom is free to roam throughout the body of the material.

In electrolytes both positive and negative ions are mobile.

Even in dry air there are enough ions to discharge a body in a few minutes.

When some charge is transferred to a conductor it readily distributes throughout the surface of the conductor (in copper – within about 10^{-12}s).

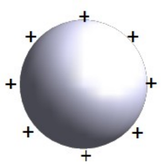


Fig.1.2 Any net charge placed on a metal sphere spreads very quickly over its surface

For a conductor, the charge always resides on the surface of the conductor. The (free) charge can not reside inside (i.e, in internal volume of) a conductor.

An **insulator** is a material which offers high resistance to the flow of electricity through it.

Examples of insulators: Non-metals like glass, porcelain, plastic, nylon, wood.

Insulators do not have free electrons. The electrons are bound with the atoms/molecules.

In an insulator such as NaCl, the valance electron in Na atom is transferred to Cl atom. The Na^+ and Cl^- ions form ionic bond in which all electrons are bound to particular atomic sites.

Insulators are also known as **dielectric** material.

When some charge is transferred to an insulator, it does not spread. It stays at the same place.

Charges can reside inside an insulator, i.e. inside the volume of the insulator.

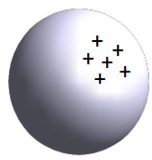


Fig.1.3 Any charge placed on an insulator does not Spread on its surface. Net charge may be found in a small region on the surface of an insulator.

Earthing

When a charged body is brought in contact with the earth, say, through a conducting wire, the excess charge flows to the earth. The process of sharing the charge with the earth is called grounding or earthing.

Charging by Induction

The process of giving equal and opposite charges to two conducting spheres (i.e., charging the spheres) by the method of induction is illustrated below.

The conducting spheres A and B are mounted on insulated stands and are initially uncharged (Fig.1.4(a)).

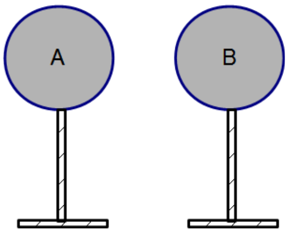


Fig.1.4(a)

Let these be kept in contact with each other and a positively charged glass rod be brought near A.

The free electrons in the conducting spheres are attracted towards the glass rod. This leaves an excess of positive charge on the rear surface of sphere B.

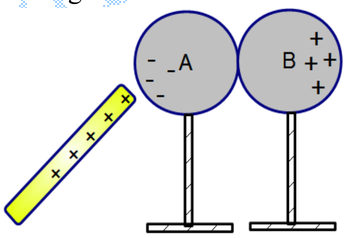


Fig.1.4(b)

However, charges can not leave the metal sphere. They, therefore, reside on the surfaces as shown in Fig.1.4(b). The left surface of sphere A has an excess of negative charge and the right surface of sphere B has an excess of positive charge.

These are called induced charges.

Now separate these spheres by a small distance while the glass rod is still held near the sphere A, as shown in Fig.1.4(c).

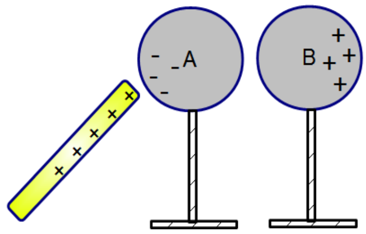


Fig.1.4(c)

Negative induced charge will reside on A and positive induced charge will reside on B. The two spheres attract each other.

Now move away the glass rod. The spheres will remain charged, as shown in Fig.1.4(d).

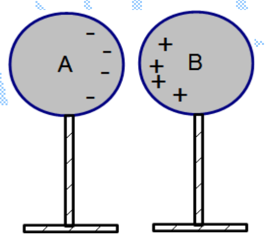


Fig.1.4(d)

Now separate the spheres quite apart. The charges on them get uniformly distributed over them, as shown in Fig.1.4(e).

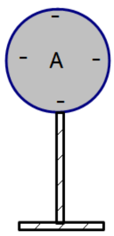
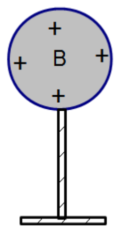


Fig.1.4(e)



This method is known as charging by induction. The glass rod did not lose any charge.

Conceptual Questions

1C1. What do you understand by electric charge? (Electro-I-1C1)

1C2. When does a neutral body acquire negative (positive) charge. (Electro-I-1C2)

1C3. What are SI unit and dimensions of electric charge? (Electro-I-1C3)

1C4. Define 1 coulomb. (Electro-I-1C4)

1C5. What is a gold leaf electroscope? (Electro-I-1C5)

1C6. Distinguish between a conductor and an insulator. (Electro-I-1C6)

1C7. What is earthing? (Electro-I-1C7)

1C8. How will you give two metal spheres equal and opposite charges by the method of induction? (Electro-I-1C8)

1C9. Can the above method of charging by induction (question 1C8) used for plastic (insulator) spheres? (Electro-I-1C9)

1C10. (a) What would happen if the rod inducing charges in the method of induction (question 1C8) were removed before the spheres are separated?

(b) Would the induced charges be equal in magnitude even if the metal spheres (question 1C8) had different sizes?

(c) How would the charge distribution change when the spheres A and B, after charging, are removed far apart? (Electro-I-1C10)

1C11. Demonstrate charging of a single metal sphere by induction.(How can you charge a metal sphere positively without touching it?) (Electro-I-1C11)

I.2(A) CONSERVATION OF CHARGE

The principle of conservation of charge states that the total charge of an isolated system always remains constant (conserved).

The principle of conservation of charge means that in an isolated system electric charges may be created or annihilated but the process will happen in such a manner that the total charge before and total charge after the process remains the same.

Examples:

(a) Pair production $\gamma = e^{+} + e^{-}$,

(b) Pair annihilation: $e^{+} + e^{-} = \gamma + \gamma$,

(c) Radioactive decays: ${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$,

(d) Neutron decay $n \rightarrow p + e^{-} + \bar{\nu}$,

(e) Chemical reaction $Na^{+} + Cl^{-} = NaCl$.

Clarification:

Conservation of charge is an independent property and does not depend on the scalar nature of the charge.

I.2(B) ADDITIVE NATURE OF CHARGES

Charge is a scalar quantity. It has a magnitude but no direction.

Charges add and subtract like numbers.

Because of positive and negative nature of charges, proper signs must be used while adding the charges in a system.

Example:

Let a system consists of three charges :

$q_1 = 20\,\mu C$, $q_2 = -15\,\mu C$ and $q_3 = 6\,\mu C$.

Then the total charge on the system is:

$20\,\mu C + (-15)\,\mu C + 6\,\mu C = 11\,\mu C$.

The additive property of the charge is due to its scalar nature.

Clarification:

(i) The value of charge remains invariant in different inertial frames of references, even if they have relative motion between them.

(ii) Many scalar quantities, such as kinetic energy, depends on the relative motion of inertial frame.

(iii) A famous relation showing variation of mass with speed is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} ,$$

where m_0 *= rest mass, v = speed and c = speed of light (a universal constant)*

I.2(C) QUANTIZATION OF CHARGE

The principle of quantization of charge states that all free charges are an integral multiple of a basic unit of electric charge denoted by e . That is the charge on a body is always,

$$q = ne$$

where n is an integer, positive or negative.

The value of the basic unit of charge, e, is (up to four significant digits)

$$e = 1.602 \times 10^{-19}\,\text{C} .$$

The basic unit of charge, e, is the magnitude of charge that an electron or proton carries.

By convention the charge on an electron is taken as negative and is written as $-e$. The charge on proton is written as $+e$ or e.

Examples:

(i) Charge on α particle = 2 e,

(ii) Charge on Cu nucleus = 29 e,

(iii) Charge on Cl^- ion = – e, etc.

Clarifications:

(i) The smallest (free) negative unit of charge is found on an electron.

(ii).The smallest (free) positive unit of charge is found on a proton.

(iii) Charge distribution may be taken as continuous when its magnitude is >> e.

Comment:

It has been established that proton and neutron have an internal structure. But an electron does not have any internal structure.

A proton consists of two u-quarks and one d-quark, while a neutron consists of two d-quarks and one u-quark. The charge of u-quark is $+\frac{2}{3}e$, and the charge of d-quark is $-\frac{1}{3}e$.

It has been established that a free quark does not exist in nature. Only two quarks or three quarks bound states exist in nature. That too in such a manner that the charge on these bound states is always an integral multiple of the basic unit e. Therefore, the statement of the principle of quantization remains true.

In all six types of quarks have been identified and named as, up (u), down (d), strange (s), charm (c), bottom (b) and top (t). These are studied in higher classes.

Clarification:

Quantization of charge is a basic law of nature.

Mass is not quantized. Length is not quantized.

In certain situations, such as for an electron in a hydrogen atom, energy and angular momentum are quantized.

It is not fully clear, why charge is quantized?

Conceptual Questions

2C1. What do you mean by conservation of charge? Illustrate your answer by suitable examples. (Electro-I-2C1)

2C2. What do you mean by quantization of charge? Illustrate your answer using suitable examples. (Electro-I-2C2)

2C3. Write basic properties of electric charge. (Electro-I-2C3)

2C4. If a body is charged by rubbing, what happens to its weight (in principle)? (Electro-I-2C4)

2C5. Three objects A, B and C, are made of different substances. When A is rubbed with B, object A becomes positively charged and B becomes negatively charged. But when A is rubbed with C, object A becomes negatively charged and C becomes positively charged. What will be the charge on objects B and C when they are rubbed against each other? (Electro-I-2C5)

Numerical Questions

2N1. If 10^{10} electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the body? (Electro-I-2N1)

2N2. A polythene piece rubbed with wool is found to have negative charge of $3\times10^{-7}\,\text{C}$. (a) Estimate the number of electrons transferred (answer from which to which?). (b) Is there a transfer of mass from wool to polythene? (Electro-I-2N2)

2N3. How many electrons are contained in –1 C of charge? What is the total mass of these electrons? (Electro-I-2N3)

2N4. A copper sphere of mass 2.0 g contains about 2×10^{22} atoms. The charge on the nucleus of each atom is 29e. What fraction of the electrons must be removed from the sphere to give it a charge of +2 μC? (Electro-I-2N4)

I.3 COULOMB'S LAW – FORCE BETWEEN TWO POINT CHARGES

The **Coulomb's law states that** the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the two charges. The force is directed along the line joining the two charges.

That is

$$F \propto \frac{q_1 q_2}{r^2}\ , \text{or}$$

$$F = \frac{k\,q_1 q_2}{r^2}\ ,$$

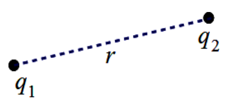


Fig.3.1

where the constant of proportionality k depends on the medium between the two charges.

For vacuum

$$k = \frac{1}{(4\pi\epsilon_0)} \quad .$$

Here ϵ_0 is called the permittivity of free space (vacuum).

The values of these constants are

$k = 9\times10^9 \ \mathrm{N\,m^2C^{-2}}$ and

$$\epsilon_0 = 8.85\times10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N\,m^2}} \quad .$$

Dimensions of *k* are: $\mathrm{M^1L^3T^{-4}A^{-2}}$.

Dimensions of ϵ_0 are: $\mathrm{M^{-1}L^{-3}T^4A^2}$.

One coulomb is that amount of charge which when placed at a distance of one meter from an identical charge in vacuum, is repelled by a force of $9\times10^9\,\mathrm{N}$.

Clarification:

The value of k is large because of the choice of units used. The choice of value of 1C *of charge is decided by the definition of ampere (the unit of electric current).*

$$1\,\mathrm{C} = 1\,\mathrm{A\cdot s} \quad .$$

Ampere shall be defined in Unit III.

(i) **Permittivity**: If the charges are placed in a medium instead of in vacuum, the Coulomb's law becomes,

$$F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \quad .$$

The constant ϵ is called the permittivity of the medium.

For a medium $\epsilon > \epsilon_0$.

The force between two charges q_1 and q_2 in the presence of medium reduces in comparison to that for vacuum (this happens due to induced charges in the medium).

(ii) **Relative permittivity**: The ratio of permittivity of a medium and permittivity of vacuum is called the relative permittivity of the medium. That is

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \quad ,$$

is the relative permittivity of the medium.

It is also called the dielectric constant (*K*) of the medium $\epsilon_r \equiv K$.

Clarifications:

(a) The permittivity relates the units of electric charge to mechanical quantities like length and force.

(b) Permittivity is the measure of how much hindrance is encountered when forming an electric field in a medium.

(c) Permittivity relates to a material's ability to transmit (or "permit") an electric field.

(iii) **Dimensions of permittivity** are

$$\mathrm{M^{-1}L^{-3}T^4A^2} \quad .$$

The relative permittivity, ϵ_r , is a dimensionless quantity.

Comments:

(i) $F_{medium} = \frac{F_{vacuum}}{\epsilon_r}$.

(ii) The dielectric constant (or relative permittivity) of a medium is the ratio of Coulomb's force of repulsion/attraction between two point charges separated by a certain distance in vacuum to the force of repulsion/attraction between the same two charges, held at the same distance apart in the medium.

$$\epsilon_r = \frac{F_{vacuum}}{F_{medium}} .$$

Clarification:

Let two charges q_1 and q_2 are separated in a medium of dielectric constant K by a distance r_{med} . The force between them is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{K r_{med}^2} . \quad (1)$$

Now let these two charges are placed in vacuum, separated by a distance r_{vac} . The force between them is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{vac}^2} . \quad (2)$$

The two forces (1) and (2) will have the same magnitude, when

$$r_{vac} = \sqrt{K}\, r_{med} .$$

This can be interpreted as follows:

"When a dielectric slab of thickness t is introduced in between two charges separated by a distance r , then the force between the charges should read

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\left(r - t + t\sqrt{K}\right)^2} ."$$

Coulomb (Charles Augustin de Coulomb, 14 Sept 1736 – 23 Aug 1806) was French. He constructed a torsion balance to measure force of repulsion between two charged spheres. He gave the inverse square law of force between two charges.

He also found the inverse square law of force between two magnetic poles.

VECTOR FORM OF COULOMB'S LAW

Let position vectors of two charges q_1 and q_2 placed at points A and B are $\vec{r}_1$ and $\vec{r}_2$, respectively.

We denote force on charge q_1 (by charge q_2) by $\vec{F}_{12}$, and force on charge q_2 (by charge q_1) by $\vec{F}_{21}$.

The vector leading from q_1 to q_2 is denoted by $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$. In the same way the vector leading from q_2 to q_1 is denoted by $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$. Their corresponding unit vectors are denoted by $\hat{r}_{21}$ and $\hat{r}_{12}$.

The Coulomb's force law between two point charges is, then, expressed as (Fig.3.2(a))

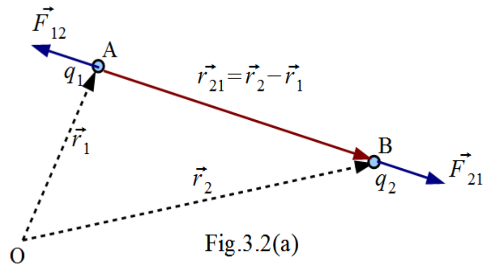


Fig.3.2(a)

$$\vec{F}_{21}=\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} \qquad ...(3.1)$$

This equation is valid for any sign of q_1 and q_2 .

If $q_1 q_2>0$ (i.e., they are of the same sign), then $\vec{F}_{21}$ is along $\hat{r}_{21}$ (Figs.3.2(a) & (b)).

If $q_1 q_2<0$ (i.e., they are of opposite sign), then $\vec{F}_{21}$ is along $-\hat{r}_{21}(\equiv\hat{r}_{12})$ (Fig.3.2(c)).

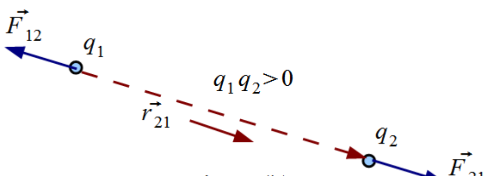


Fig.3.2(b)

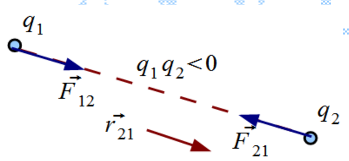


Fig.3.2(c)

The force $\vec{F}_{12}$ on charge q_1 due to charge q_2 , is obtained from Eq.(3.1):

$$\vec{F}_{12}=\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}=-\vec{F}_{21} \quad .$$

Thus Coulomb's law agrees with the Newton's third law.

Clarifications:

(i) The Eq.(3.1) gives the force between two charges placed in vacuum. If the charges are placed in matter or the intervening space has matter, the situation gets modified. The ϵ_0 is then replaced by $\epsilon_r\epsilon_0$.

(ii) There exists four types of fundamental forces in nature.

(I) Strong force,

(II) Electromagnetic force,

(III) Weak force, and

(IV) Gravitational force.

The electromagnetic and gravitational, both are inverse square laws forces, and are of long range.

The strong force is the strongest, about 100 times stronger than the electromagnetic force.

The weak force is about 1/1000 to 1/billion times weaker than the electromagnetic force.

The strong and weak forces are both short range forces.

Electric and magnetic forces (electromagnetic forces) determine the properties of atoms, molecules and bulk matter.

RATIO OF ELECTRIC FORCE AND GRAVITATIONAL FORCE

Let two point particles having charges q_1 and q_2 , and masses m_1 and m_2 , respectively, are separated by a distance *r*. Then the magnitudes of the electrostatic force and the gravitational force between the two particles, are, respectively,

$$F_E = \frac{k q_1 q_2}{r^2} \text{ , and}$$

$$F_G = \frac{G m_1 m_2}{r^2} \text{ ,}$$

where

$$k = \frac{1}{4\pi\epsilon_0} \text{ .}$$

Therefore, the ratio is

$$\frac{F_E}{F_G} = \frac{k q_1 q_2}{G m_1 m_2} \text{ .}$$

(I) For a proton and an electron (substitute the values and solve to find)

$$\frac{F_E}{F_G} = 2.4\times 10^{39} \text{ .}$$

(II) For a proton and another proton (substitute the values and solve to find)

$$\frac{F_E}{F_G} = 1.2\times 10^{36} \text{ .}$$

Thus, **the electric force is much stronger than the gravitational force**.

Conceptual Questions

3C1. State Coulomb's law. (Electro-I-3C1)

3C2. What are the dimensions of k and ϵ_0 ? (Electro-I-3C2)

3C3. What is (i) permittivity, (ii) relative permittivity, of a medium? (Electro-I-3C3)

3C4. What are the dimensions of permittivity and relative permittivity? (Electro-I-3C4)

3C5. Can you apply the expression $F = k q_1 q_2 / r^2$ for charges distributed on irregular shaped bodies (Fig.3C5)?

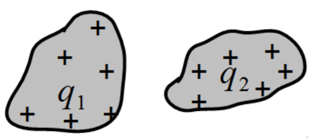


Fig. 3C5

(Electro-I-3C5)

3C6. If charge is uniformly distributed over spherical surfaces (bodies), then can one use the relation $F = k q_1 q_2 / r^2$ for evaluating the force?(see Fig.3C6)(Electro-I-3C6)

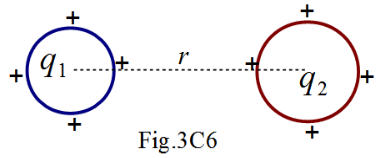


Fig.3C6

3C7. Is Coulomb's law (as expression of force between two charges) valid if charges are moving? (Electro-I-3C7)

3C8. Is Coulomb's law (as expression of force between two charges) valid for $r \to 0$? (Electro-I-3C8)

3C9. Express Coulomb's law in vector form, explaining the meaning of various terms used. (Electro-I-3C9)

3C10. A charge $+Q$ is placed at the origin of a Cartesian coordinate system. Another charge $+q$ is placed at the position vector $\vec{r}$. Express the Coulomb's law for this system of charges in vector form. (Electro-I-3C10)

3C11. Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two point masses, both have inverse square dependence on the distance between the charges/masses.

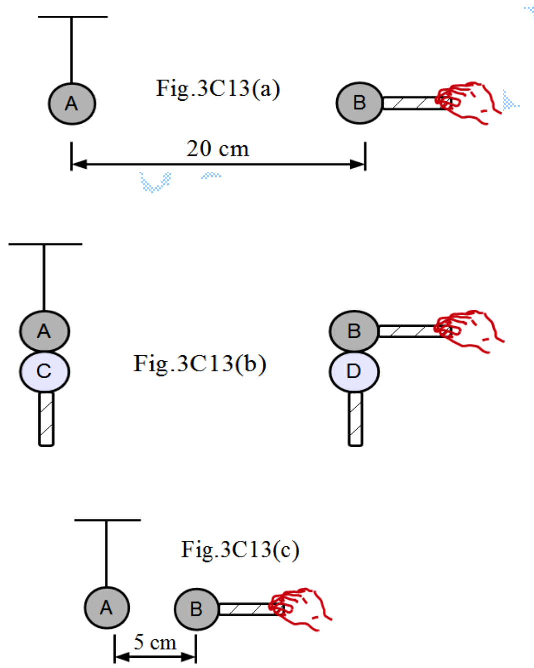
Compare the strength of the electric force and gravitational force for (I) an electron and a proton, and (II) two protons. (Electro-I-3C11)

3C12. What are the dimensions of the ratio

$e^2/(4\pi\epsilon_0 G m_e m_p)$? (Electro-I-3C12)

3C13. A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating handle is brought close to A such that the distance between their centers is 20 cm, as shown in Fig.(a). The resulting repulsion of A is noted. Let it be F. Now the spheres A and B are touched by uncharged metallic spheres C and D, respectively, as shown in Fig.(b). C and D are then removed and B is brought closer to A to a distance of 5.0 cm between their centers, as shown in Fig.(c). What is the expected repulsion of A on the basis of Coulomb's law. Assume all spheres have identical sizes. Ignore the sizes of the spheres in comparison to the distances involved.

(Electro-I-3C13)



3C14. What effect does an isotropic homogeneous medium have on Coulomb's law for charges embedded in it. (Electro-I-3C14)

3C15. Plot a graph showing the variation of Coulomb's force (F) versus $\left(\frac{1}{r^2}\right)$ where r is the distance between the two charges of each pair of charges: (1μC, 2μC) and (2μC, $-3\,\mu C$). Interpret the graphs obtained. (Electro-I-3C15)

3C16. Plot a graph showing the variation of Coulomb's force (F) versus r, where r is the distance between the two charges of each pair of charges: (1μC, 2μC) and (2μC, $-3\,\mu C$). (Electro-I-3C16)

Numerical Questions

3N1. What is the force between two small charged spheres having charges of $0.3\,\mu C$ and $0.4\,\mu C$ placed 30 cm apart in air? (Electro-I-3N1)

3N2. The magnitude of electrostatic force on a small sphere of charge $-0.8\,\mu C$ due to another small sphere of charge $1.6\,\mu C$ in air is 0.8N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first? (Electro-I-3N2)

3N3. Estimate the accelerations of an electron and a proton due to the electric force of their mutual attraction when they are $1\,\text{Å}\,(=10^{-10}\,m)$ apart. Given $m_p=1.67\times10^{-27}\,kg$, and $m_e=9.11\times10^{-31}\,kg$. (Electro-I-3N3)

3N4. A cup contains 180 g of water. What is the total charge on the electrons available in this water? (Electro-I-3N4)

3N5. Two small charged spheres of copper, having mass 1 kg each, have charges 200 nC and 300 nC respectively. They are placed 30 cm apart in air. What is (i) the gravitational force between the two spheres, (ii) the electrostatic force between the two spheres. (Electro-I-3N5)

3N6. Two identical conducting charged spheres A and B have charges, Q and $3Q$ are separated by a distance R. The force of repulsion is F. Now these sphere are brought in contact with each other and then separated back to distance R. What is the force of repulsion now? (Electro-I-3N6)

3N7. A charge Q is divided into two parts q, and $(Q-q)$. These are then separated by a distance R. If the force of repulsion between them is maximum, what is the value of q in terms of Q?

(Electro-I-3N7)

3N8. (a) Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion, if the charge on each is $0.65\,\mu C$? The radii of A and B are negligible compared to the distance of separation.

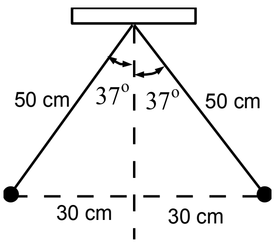
(b) What is the force of repulsion, if each sphere is charged double the above amount, and the distance between them is halved? (Electro-I-3N8)

3N9. Suppose the spheres A and B in question 3N8 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B? (Electro-I-3N9)

3N10. Two point charges q_1 and q_2 are 3 cm apart, and their combined charge is 20 μC. (a) If one repels the other with a force of 0.075 N, what are the two charges? (b) If one attracts the other with a force of 0.525 N, what are the magnitudes of the two charges? (Electro-I-3N10)

3N11. Two identical tiny metal balls carry charges of +3 nC and – 12 nC. They are 3 cm apart. (a) Compute the force of attraction. (b) The balls are now touched together and then separated to 3 cm. Describe the forces on them now.(Electro-I-3N11)

3N12. Two balls shown in Fig. have identical masses of 0.20 g each. When suspended from 50-cm long strings, they make an angle of 37^o to the vertical. If the charges on each are the same, how large is each charge? (Electro-I-3N12)



3N13. Two positive point charges a distance b apart have sum Q. For what values of the charges is the Coulomb force between them a maximum?

(Electro-I-3N13)

I.4 (A) FORCES BETWEEN MULTIPLE CHARGES:

Principle of superposition

The principle of superposition states,

"Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected by the presence of other charges."

Consider three charges, q_1 , q_2 and q_3 located at position vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$, respectively. The force on q_3 by q_1 is $\vec{F}_{31}$, and the force on q_3 by q_2 is $\vec{F}_{32}$, as shown in Fig.4.1. These are

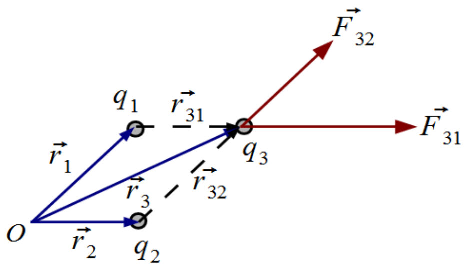


Fig.4.1

$$\vec{F}_{31}=\frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} \quad \text{, and}$$

$$\vec{F}_{32}=\frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_{32}^2}\hat{r}_{32} \quad .$$

The resultant force is

$$\vec{F}_3=\vec{F}_{31}+\vec{F}_{32} \quad .$$

For convenience, we put $\vec{F}_1=\vec{F}_{31}$, $\vec{F}_2=\vec{F}_{32}$, $\vec{F}=\vec{F}_3$ and use parallelogram law of vector addition (see Fig.4.2)). We find the magnitude of the resultant force to be

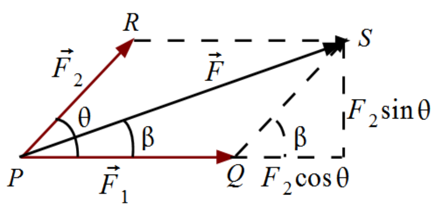


Fig.4.2

$$F=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta} \quad .$$

The direction is determined from the relation

$$\tan\beta=\frac{F_2\sin\theta}{F_1+F_2\cos\theta} \quad .$$

Clarification:

The principle of superposition means that there are no additional three body forces or four body forces, etc., when many charges are present in a region.

I.4(B) SUPERPOSITION PRINCIPLE AND CONTINUOUS CHARGE DISTRIBUTION

Comment: All electrostatics is basically a consequence of Coulomb's law and the superposition principle.

Principle of superposition: Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time.

Continuous charge distributions

(i) **Line charge distribution**

Imagine that charge Q is uniformly distributed along a line of length L. Then, the linear charge density is defined by the relation:

$$\lambda=\frac{Q}{L} \quad .$$

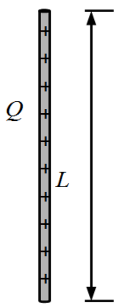


Fig4.3(a)

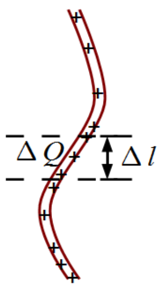


Fig.4.3(b)

If the distribution is not uniform, one defines local linear charge density by the relation

$$\lambda = \lim_{\Delta l \to 0} \left(\frac{\Delta Q}{\Delta l}\right) \quad .$$

The unit of linear charge density is coulomb per meter (C/m).

(ii) **Surface charge distribution**

Let a charge Q is uniformly distributed over a surface of area A. Then, the surface charge density is defined by the relation

$$\sigma = \frac{Q}{A} \quad .$$

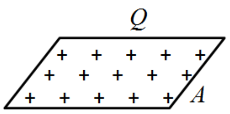


Fig.4.4(a)

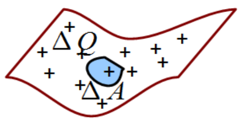


Fig.4.4(b)

For a non-uniform surface charge distribution, the local surface charge density is defined by the relation

$$\sigma = \lim_{\Delta A \to 0} \left(\frac{\Delta Q}{\Delta A}\right) \quad .$$

The unit of surface charge density is coulomb per square meter (C/m^2) .

(iii) **Volume charge distribution:**

Let a charge Q is uniformly distributed in a volume V. Then, the volume charge density is defined by the relation

$$\rho = \frac{Q}{V} \quad .$$

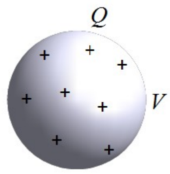


Fig.4.5

For a non-uniform volume charge distribution, the local volume charge density is defined by the relation

$$\rho = \lim_{\Delta V \to 0} \left(\frac{\Delta Q}{\Delta V}\right) \quad .$$

The unit of volume charge density is coulomb per cubic meter (C/m^3) .

(Comment: At microscopic level, charge distribution is discontinuous.)

Conceptual Questions

4C1. State Principle of superposition for electrostatic forces. (Electro-I-4C1)

4C2. Consider three charges q_1, q_2, q_3 at positions specified by position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. Obtain an expression for the resultant force on charge q_3 . (Electro-I-4C2)

4C3. When a charged comb is brought near a small piece of paper, it attracts the piece. Why? (Electro-I-4C3)

4C4. Two charges q_1 and q_2 are placed at a distance of d . The force exerted by q_1 on is F_{21} . Now a third charge is place very near q_1 . How will the force F_{21} effected by the presence of the third charge? (Electro-I-4C4)

4C5. A charge q is situated at the center of the line joining two equal and like charges Q. This system is in equilibrium. What is the value of q in terms of Q? (Electro-I-4C5)

Numerical Questions

4N1. Three equal charges, each Q, are placed on the vertices of an equilateral triangle of side "a". What is the net force on any charge due to the other two charges? (Electro-I-4N1)

4N2. Consider three charges, each equal to q at the vertices of an equilateral triangle of side "l". What is the force on a fourth charge Q placed at the centroid of the triangle? (Electro-I-4N2)

4N3. Four equal charges (each q) are placed on the corners of a square with side "a". What is the resultant force on any one of the charges, due to the other three charges? (Electro-I-4N3)

4N4. Two like charges q_1 and q_2 are fixed and separated by a distance R. A third charge q is placed between the two charges so that the resultant force on it due to the two charges becomes zero. What are the distances of the third charge q from q_1 and q_2 . (Electro-I-4N4)

4N5. In the above question 4N4 if the two like charges q_1 and q_2 are free and we want to place the third charge q in such a manner that the system of charges remains in equilibrium (may be unstable), then show that the third charge must be of opposite nature and of magnitude

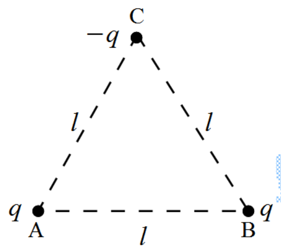
$$q=\frac{q_1 q_2}{(\sqrt{q_1}+\sqrt{q_2})^2} .$$

(Electro-I-4N5)

4N6. Four point charges $q_A=2\,\mu C$, $q_B=-5\,\mu C$, $q_C=2\,\mu C$ and $q_D=-5\,\mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on the charge of $1\,\mu C$ placed at the center of the square? (Electro-I-4N6)

4N7. Two unlike charges q_1 and $-q_2$ are fixed and separated by a distance R. It is given that $|q_1|>|-q_2|$. A third charge q is placed somewhere on the line joining the two charges, so that the resultant force on it due to the two unlike charges is zero. What are the distances of the third charge from q_1 and $-q_2$? (Electro-I-4N7)

4N8. Consider the charges q, q, $-q$, placed at the vertices of an equilateral triangle, as shown in Fig (below). What is the force on each charge? (Electro-I-4N8)



4N9. Two point charges $q_A=3\,\mu C$ and $q_B=-3\,\mu C$ are located 20 cm apart in vacuum. If a negative test charge of magnitude 1.5 nC is placed at the middle point, what is the force experienced by this test charge? (Electro-I-4N9)

I.5 (A) ELECTRIC FIELD

The concept of electric field was introduced by Michael Faraday.

An electric charge creates an electric field in the space around it, in which when any other charge is placed, it experiences an electrostatic force of repulsion or attraction.

The charge which is producing the electric field, is called a source charge and any other charge, which tests the effect of a source charge, is called a test charge.

The electric field intensity (or the electric field strength) at any point is defined as numerically equal to the force experienced by a unit positive test charge placed at that point:

$$\vec{E}=\frac{\vec{F}}{q_0} .$$

where q_0 is a test charge and $\vec{F}$ is the force on the test charge.

The direction of $\vec{E}$ is that of the force experienced by the positive test charge.

The test charge must be small enough not to disturb the charges that create the electric field being measured:

$$\vec{E}=\lim_{q_0\to 0}\frac{\vec{F}}{q_0} .$$

The **SI unit of the electric field** strength is newton per coulomb $(\mathrm{N\,C^{-1}})$.

Another equivalent unit of electric field is volt per meter (V/m).

The **dimensions of the electric field** are $\mathrm{M^1 L^1 T^{-3} A^{-1}}$.

The electric field is a vector quantity.

Clarifications:

(i) The field strength $\vec{E}$ is a property of a point in space and depends only on the source charge(s) of the field. The field exists whether or not we choose to examine it with a test charge. The field exists at every point in the three dimensional space.

(ii) The value of $\vec{E}$ does not depend on the magnitude of the test charge.

(iii) The electric field due to discrete charge is not defined at the location of the discrete charge.

Comment:

A charge "q" placed in an electric field $\vec{E}$ experiences a force ,

$$\vec{F}=q\vec{E} .$$

I.5(B) ELECTRIC FIELD LINES

Electric field lines are a way of pictorially mapping the electric field around a configuration of charges.

In general, an electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point. An arrow on the curve specifies the direction of the field.

A field line is a space curve, i.e., a curve in three dimensions.

Properties of electric field lines

(i) Electric field line (or line of force) is an imaginary line along which a positive test charge will tend to move if left free.

(ii) The direction of the field at a point is along the tangent to the line of force.

(iii) Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.

(iv) In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.

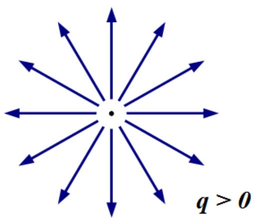


Fig.5.1(a)

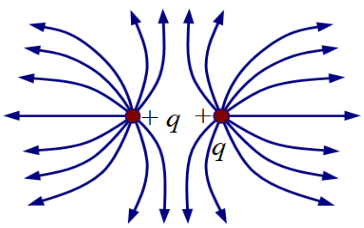


Fig.5.1(b)

(v) Two field lines can never cross each other. (If they did, the field at the point of intersection will not have a unique direction, which is absurd.).

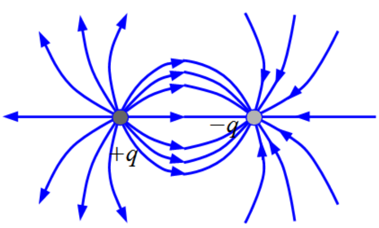


Fig.5.1(c)

(vi) Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field.

(vii) Field lines do not exist inside a conductor.

(viii) Density of field lines is proportional to the strength of the electric field. (Density of lines means the number of lines of force per unit area intercepted by a surface normal to the field.)

Physical significance of electric field

The electric field is a convenient way of describing action at a distance between two interacting charges.

The true significance emerges when we consider the force between two distant charges q_1 and q_2 in accelerated motion.

The greatest speed with which a signal or information can go from one point to another is c, the speed of light.

Thus the effect of any motion of q_1 on q_2 cannot arise instantaneously. There will be some time delay between the effect (force on q_2) and the cause (motion of q_1).

It is here that the notion of electric field (strictly, electromagnetic field) is useful. The field picture is this:

"The accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed c, reach q_2 and cause a force on q_2. The notion of field accounts for the time delay."

The electric and magnetic fields are detected by their effects (forces) on charges. These are now regarded as physical entities, not merely mathematical constructs. They have their laws (known as Maxwell equations). They can transport energy in the form of EM waves. (We shall study EM waves in Unit V.)

I.5(C) ELECTRIC FIELD DUE TO A POINT CHARGE

Let a charge Q is located at point O (origin). We wish to determine the electric field due to Q at a point P whose position vector is $\vec{r}$. For this we place a test charge q_0 at P and measure the Coulomb's force on it (Fig.5.2(a)):

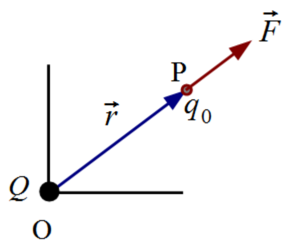


Fig.5.2(a)

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q\,q_0}{r^2}\,\hat{r} \;.$$

The electric field $\vec{E}$ at point P is, then, equal to the force experienced per unit test charge. That is,

$$\vec{E} = \frac{\vec{F}}{q_0} \;,$$

or $$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,\hat{r} \;.$$

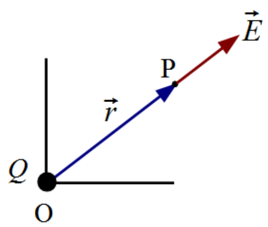


Fig.5.2(b)

The electric field strength at a point is (i) directly proportional to the magnitude of the source charge and (ii) inversely proportional to the square of the distance. It is directed along the line joining the source charge to the point (Fig.5.2(b)).

Comments:

(i) For a positive charge, the electric field vector is directed radially outward from the charge.

(ii) For a negative charge, the electric field vector, at each point, points radially inward.

(iii)The Fig.5.3 shown below depicts the electric field vectors due to a point charge (Q >0) at

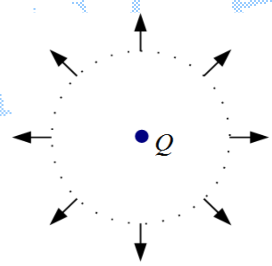


Fig.5.3

equidistant points in various directions. At equal distances from charge Q, the magnitude of the electric field $\vec{E}$ is the same. Thus the magnitude of the electric field at points that lie on a sphere with the point charge Q at its center, is the same. That is the electric field due to a point charge has a spherical symmetry.

I.5(D) ELECTRIC FIELD DUE TO A SYSTEM OF CHARGES (PRINCIPLE OF SUPERPOSITION)

Consider a system of two charges q_1 and q_2 .

We wish to determine the net electric field at a point P.

The position vector of P with respect to q_1 is $\vec{r}_{1P}$ and the position vector of P with respect to q_2 is $\vec{r}_{2P}$.

The electric field at P is, by definition, the force experienced per unit charge by a test charge q_0 placed at P. Therefore, the force on the test charge is, (see Fig.5.4),

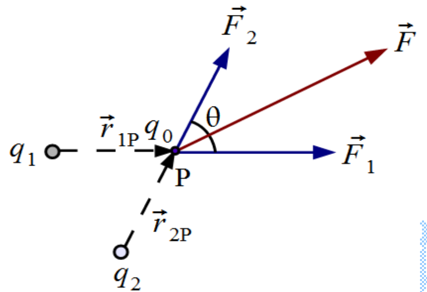


Fig.5.4

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_0}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_0}{r_{2P}^2}\hat{r}_{2P} .$$

The electric field is (from $\vec{E} = \frac{\vec{F}}{q_0}$)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P} ,$$

or $\vec{E} = \vec{E}_1 + \vec{E}_2$.

This is the **principle of superposition of electric fields:**

"The electric field at any point P due to a system of charges is the vector sum of the electric fields at that point due to individual charges taken one at a time. The individual electric field by a charge is unaffected by the presence of other charges."

The vector addition of two electric fields $\vec{E}_1$ and $\vec{E}_2$ is illustrated in Fig.5.5 below:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 .$$

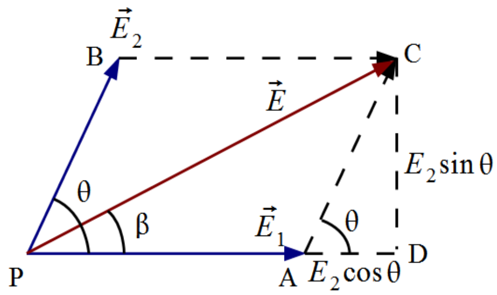


Fig.5.5

The magnitude of the resultant field is

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta} .$$

The direction is determined from the relation

$$\tan\beta = \frac{E_2 \sin\theta}{E_1 + E_2\cos\theta} .$$

I.5(E) RING CHARGE DISTRIBUTION

Consider a uniformly charged ring of radius *a*.

The linear charge density is λ . The total charge on the ring is

$$Q = 2\pi a\lambda .$$

A point P is situated on the axis at a distance *x* from the center O of the ring.

Consider a small element ACB of the ring. Let the length of the element is AB= dl , C is the center of the element. The charge on the line element is $dq = \lambda\, dl$. The electric field intensity at P due to this charge element is

$$|d\vec{E}| = k\,\frac{dq}{r^2} \quad ,$$

where $k = 1/(4\pi\epsilon_0)$ and $r = \sqrt{x^2 + a^2}$ is the distance CP.

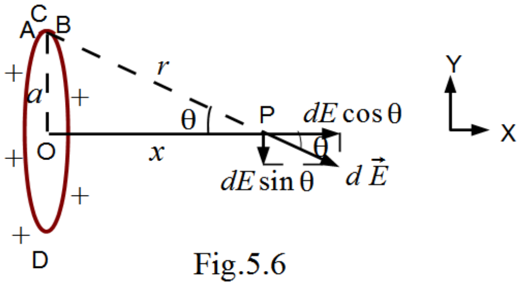


Fig.5.6

Now resolve $d\vec{E}$ into horizontal and vertical components;

$$dE_X = dE\cos\theta$$

and

$$dE_Y = -dE\sin\theta \quad .$$

Now consider a diametrically opposite element at D and of the same length. Then the contribution of this element to the electric field will be

$$dE'_X = dE\cos\theta \quad ,$$

$$dE'_Y = dE\sin\theta \quad .$$

We notice that the Y-components cancel and the X-components add.

If we consider the ring to be made up of a large number of such pair of elements, then for all pairs, only the horizontal components of the field contribute.

Hence the resultant electric field at point P due to the entire ring is

$$E = \sum_{all\ elements} dE\cos\theta = \sum_{all\ elements} \frac{k\,dq}{r^2}\cos\theta \quad .$$

Using

$$dq = \lambda\, dl \quad ,$$

$$\lambda = Q/(2\pi a) \quad ,$$

$$r = \sqrt{x^2 + a^2} \quad ,$$

$$k = 1/(4\pi\epsilon_0) \quad \text{and}$$

$$\cos\theta = x/\sqrt{x^2 + a^2} \quad ,$$

we find

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{2\pi a}\,\frac{x}{(x^2+a^2)^{3/2}} \sum_{all\ elements} dl \quad .$$

Since

$$\sum dl = 2\pi a \quad ,$$

we get

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{Qx}{(x^2+a^2)^{3/2}} \quad .$$

The electric field is axial. It points along OP direction (axis of the ring).

Limiting cases

(i) If the point P is very far away from the center of the ring, i.e., $x \gg a$, then

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{x^2} \quad .$$

The ring charge behaves as if the entire charge is concentrated at the center of the ring.

(ii) If the point P is very close to the center of the ring, i.e., $x \ll a$, then

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{a^3}\,x \quad .$$

The electric field is proportional to the distance of the point P from the center of the ring.

Comment:

An electron placed near the center of the ring charge experiences a force

$$F=-eE \quad ,$$

$$F=-\frac{eQ}{4\pi\epsilon_0 a^3}x$$

or

$$F\propto -x \quad .$$

The force is proportional to the displacement and is directed opposite to the displacement. Therefore the electron will execute SHM. The force constant for the motion is

$$K=\frac{eQ}{4\pi\epsilon_0 a^3} \quad ,$$

and the angular frequency of the motion would be

$$\omega=\sqrt{\frac{eQ}{4\pi\epsilon_0 m a^3}} \quad .$$

Conceptual Questions

5C1. What do you understand by an electric field. (Electro-I-5C1)

5C2. Define electric field strength at a given point. How is its direction assigned? (Electro-I-5C2)

5C3. Write SI unit and dimensions of electric field. (Electro-I-5C3)

5C4. What are electric field lines. Write properties of electric field lines. (Electro-I-5C4)

5C5. What is the significance of the concept of electric field? (Electro-I-5C5)

5C6. A point charge Q is located at the origin of a coordinate system. What is the electric field strength at position vector $\vec{r}$?(Electro-I-5C6)

5C7. Consider two charges q_1, q_2 at positions specified by position vectors $\vec{r}_1, \vec{r}_2$. Obtain an expression for the resultant electric field at a point P whose position vector is $\vec{r}$. (Electro-I-5C7)

5C8. Consider a ring charge distribution. Obtain an expression for the electric field on an axial point at a distance x from the center of the ring. (Electro-I-5C8)

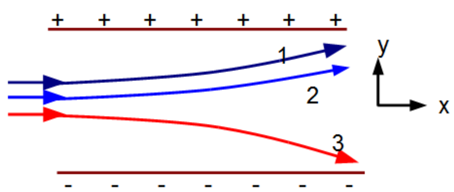
5C9. Is the magnitude of field strength constant along a line of force? (Electro-I-5C9)

5C10. A test charge is released in the field due to two point charges. Do the field lines indicate the possible paths traveled by the test charge? (Electro-I-5C10)

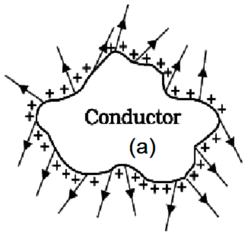
5C11. Is there any case in which a test charge would travel along the field lines? (Electro-I-5C11)

5C12. If number of lines of force emanating from a point charge are assumed to be proportional to the amount of charge Q, and strength of the electric field at some point (at a distance r from the charge) is assumed to be proportional to the number of lines crossing a unit (perpendicular) area, then show that $E\propto 1/r^2$. (Electro-I-5C12)

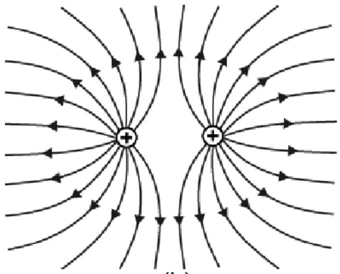
5C13. Fig. below shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has highest charge to mass ratio? (Electro-I-5C13)

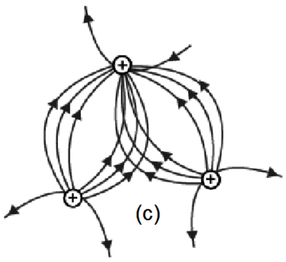


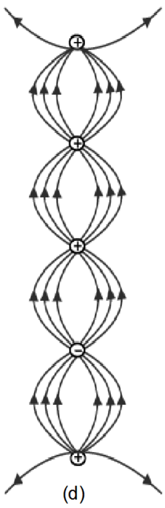
5C14. Look at the electric field line patterns drawn by a student in Fig.(a) to (d), and (e) below. Which among the curves cannot possibly represent electrostatic field? (Electro-I-5C14)

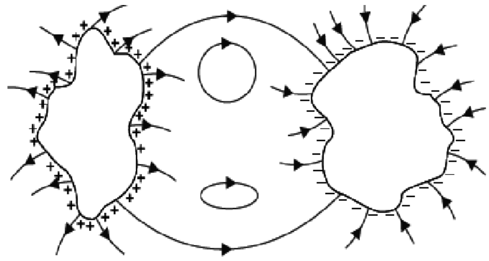


(a)


(b)


(c)


(d)


(e)

5C15. Consider a simple configuration of two charges of the same magnitude and sign placed a certain distance apart. Draw representative electric field lines and show that the equilibrium of the test charge placed at the null point is necessarily unstable. (Electro-I-5C15)

5C16. Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where E=0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable. (Electro-I-5C16)

5C17. Justify why two electric field lines never cross each other. (Electro-I-5C17)

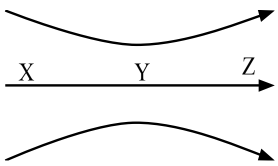
5C18. If N electric field lines emanate from a charge Q, how many electric field lines will emanate from 2Q charge. (Electro-I-5C18)

5C19. Draw a qualitative picture showing electric field pattern for a system of two charges $+Q$ and $-3Q$ separated by a small distance.(Electro-I-5C19)

5C20. The electric field lines in a certain charge free region is shown in the diagram below (see next column). Introduce the sign = or > or < in the boxes at appropriate places in the relations

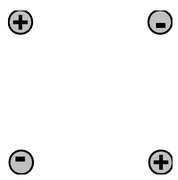
E_X ☐ E_Y ☐ E_Z , and

E_X ☐ E_Z



(Electro-I-5C20)

5C21. Draw electric field lines (qualitatively) for the quadrupole shown here and indicate the position of the electrostatic neutral point. (Electro-I-5C21)



5C22. Give arguments to justify that a point of stable equilibrium is not possible in any electrostatic field configuration. (Electro-I-5C22)

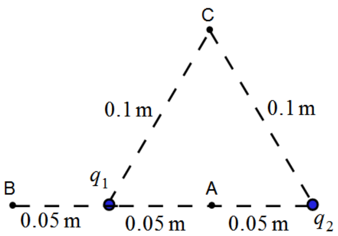
Numerical Questions:

5N1. A point charge q= 2μC experiences a force $\vec{F}=5\times10^{-5}\hat{i}$ N . (a) What is the field strength E ? (b) What happens to $\vec{E}$ if the test charge q is changed to 1 μC? (Electro-I-5N1)

5N2. Two positive charges Q_1 and Q_2 are placed on a line at a separation of R. What is the position of the null point (i.e., the point where the electric field is zero)? (Electro-I-5N2)

5N3. Two unlike charges Q_1 and $-Q_2$ ($|Q_1|>|-Q_2|$) are placed on a line at a separation of R. What is the position of the null point (i.e., the point where the electric field is zero)? (Electro-I-5N3)

5N4. Two point charges $q_1=+10^{-8}\mathrm{C}$ and $q_2=-10^{-8}\mathrm{C}$ are placed 0.1 m apart. Calculate the electric fields at points A, B, and C shown in the figure. (Electro-I-5N4)



5N5. Two point charges $q_A=3\,\mu\mathrm{C}$ and $q_B=-3\,\mu\mathrm{C}$ are located 20 cm apart in vacuum. (a) What is the electric field at the midpoint O of the line AB joining the two charges? (b) If a negative test charge of magnitude $1.5\times10^{-9}\mathrm{C}$ is placed at the middle point O, what is the force experienced by this test charge? (Electro-I-5N5)

I.6 (A) MOTION OF A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

(I) When initial velocity of charged particle is parallel to the electric field

Let a particle of charge q and mass m , moving with an initial velocity $\vec{u}$, enters a uniform electric field $\vec{E}$ (Fig.6.1). The $\vec{u}$ is parallel to $\vec{E}$.

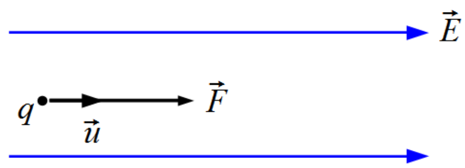


Fig.6.1

In the presence of an electric field $\vec{E}$, the charge experiences a force

$$\vec{F}=q\vec{E} .$$

As a result its motion is accelerated. The acceleration is

$$\vec{a}=\frac{q\vec{E}}{m} .$$

The field is uniform and constant, therefore, the acceleration is also uniform and constant.

The velocity and displacement of the particle at a later time t are, then, determined from the following relations:

$$\vec{v}=\vec{u}+\vec{a}\,t \quad ,$$
$$\vec{s}=\vec{u}\,t+(1/2)\vec{a}\,t^2 \quad .$$

These relations also imply:

$$v^2=u^2+2a\,s \quad .$$

(II) When initial velocity of charged particle is perpendicular to the electric field

Let a particle of charge q and mass m , moving with an initial velocity $\vec{u}=u\,\hat{i}$, directed along x-axis, enters a uniform electric field $\vec{E}=E\,\hat{j}$ directed along y-axis. The $\vec{u}$ is perpendicular $\vec{E}$.

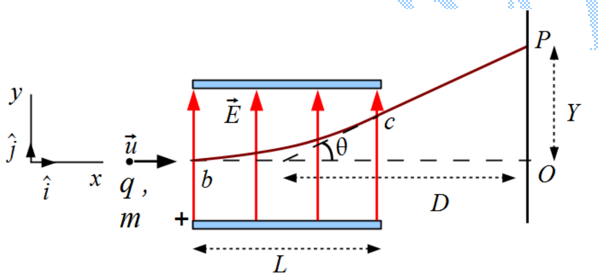


Fig.6.2

In the presence of an electric field $\vec{E}$, the charge experiences a force

$$\vec{F}=q\,\vec{E} \quad . \qquad ...(6.1)$$

As a result its motion is accelerated. The acceleration is

$$\vec{a}=\frac{q\,\vec{E}}{m} \quad . \qquad ...(6.2)$$

The field is uniform and constant, therefore, the acceleration is also uniform and constant.

For convenience, we resolve the motion in x- and y- components.

The x- and y- components of the force and the acceleration are

$$F_x=0\,, \quad a_x=0 \quad \text{and}$$
$$F_y=q\,E\,, \quad a_y=\frac{q\,E}{m} \quad . \qquad ...(6.3)$$

Since there is no force in the x-direction, the x-component of the velocity remains u, i.e.,

$$V_x = u \quad \text{at all times.} \qquad ...(6.4)$$

The y-component of particle's velocity depends on time. At time t (after it enters the electric field) it is

$$V_y = \frac{q\,E\,t}{m} \quad . \qquad ...(6.5)$$

The displacement components are

$$x = u\,t \quad \text{and} \quad y = \frac{q\,E\,t^2}{m} \quad . \qquad ...(6.6)$$

Eliminating variable t, we find

$$y = \frac{1}{2}\,\frac{q\,E}{mu^2}\,x^2 \quad . \qquad ...(6.7)$$

This is equation of a parabola.

Thus if a charged particle enters an electric field with its initial velocity perpendicular to the electric field, it follows a parabolic path.

The time spent in the electric field (length L) is

$$t' = \frac{L}{u} \quad . \qquad ...(6.8)$$

As such, the *y*-coordinate of the particle when it leaves the field region is

$$y' = \frac{qEL^2}{2mu^2} \quad . \qquad ...(6.9)$$

The *y*-component of the particle velocity when it leaves the field is

$$V_y = \frac{qEL}{mu} \quad . \qquad ...(6.10)$$

The resultant velocity of the particle when it leaves the field region is

$$V = \sqrt{u^2 + \left(\frac{qEL}{mu}\right)^2} \quad . \qquad ...(6.11)$$

The angle at which the particle emerges is such that,

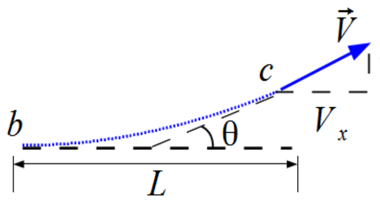


Fig.6.3

$$\tan\theta = \frac{V_y}{V_x} = \frac{qEL}{mu^2} \quad . \qquad ..(6.12)$$

If a screen is placed at a distance *D* from the centre of the field region, then, the particle hits the screen at a vertical distance,

$$Y = D\tan\theta = \frac{qELD}{mu^2}$$
$$= \frac{qELD}{2K} \quad . \qquad (6.13)$$

I.6(B). MILLIKAN'S OIL DROP EXPERIMENT

The charge of an electron was first measured by R. A. Millikan during 1909 – 1913.

In Millikan experiment, oil is sprayed in very fine drops into the space between two parallel horizontal plates separated by a distance *d*. A potential difference *V* is maintained to produce a uniform electric field *E* between the plates.

In this experiment, the drops are first held motionless by the application of a uniform field *E*. In this situation

$$qE = m_{eff}\, g \quad ,$$

where the effective mass of the drop is

$$m_{eff} = (4/3)\pi r^3(\rho - \rho_A) \quad ,$$

where ρ is the density of the drop and ρ_A is

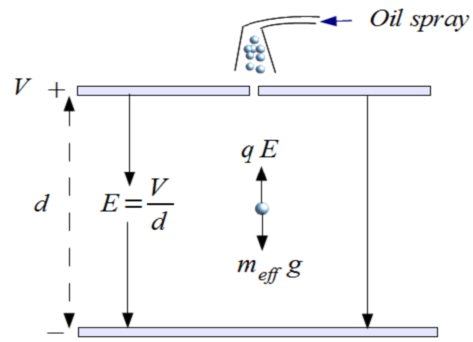


Fig.6.4

the density of the air. The air has a buoyant effect.

Therefore,

$$qE = \frac{4}{3}\pi r^3(\rho - \rho_A)\, g \qquad ...(6.14)$$

Next, the field is switched off and the drops are allowed to fall in air until they reach the terminal speed v_T .

The fluid resistance is given by the Stokes law, $F=6\pi\eta r v_T$, where η is coefficient of viscosity of air and *r* is the radius of the drop.

The condition for falling at the terminal speed is

$$6\pi\eta r v_T = m_{eff}\, g \quad ,$$

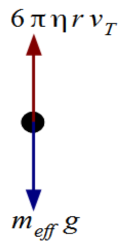


Fig.6.5

or

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3(\rho-\rho_A)g \quad . \qquad ...(6.15)$$

The radius of the drop can not be measured.

This is evaluated from Eq.(6.15),

$$r=\sqrt{\frac{9}{2}\frac{\eta v_T}{(\rho-\rho_A)g}} \quad . \qquad ...(6.16)$$

Using Eq.(6.16) in Eq.(6.14), we find the expression for the charge on a drop in terms of measurable variables (defined in the passage). This expression is

$$q=\frac{18\pi}{E}\sqrt{\frac{\eta^3 v_T^3}{2(\rho-\rho_A)g}} \quad .$$

Conceptual Questions

6C1. A particle of charge *q* and mass *m* enters a uniform and constant electric field $\vec{E}$ with initial velocity $\vec{u}$ parallel to the field. Write expressions for acceleration, velocity and displacement of the particle at a later time *t*.

(Electro-I-6C1)

6C2. A particle of charge *q* and mass *m* enters a uniform and constant electric field $\vec{E}$ with initial velocity $\vec{u}$ perpendicular to the field. (a) What are the components of the displacement and velocity at any time *t* after the particle enters the field? (b) What is the shape of the trajectory of the particle? (c) What is the angle at which the particle leaves the field. (d) What is the Y-coordinate at which the particle strikes the screen.

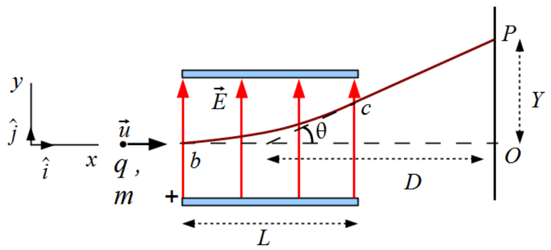


Fig.6C2

(Electro-I-6C2)

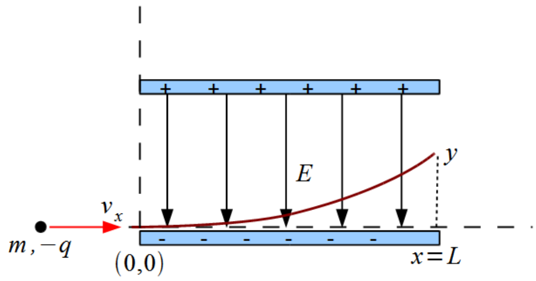
6C3. Summarize Millikan's oil drop experiment.

(Electro-I-6C3)

Numerical Questions

6N1. A particle of mass *m* and charge (–*q*), enters the region between the two charged plates, initially moving along the x-axis with speed v_x . See figure below. The length of the plate is *L* and a uniform electric field *E* is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2mv_x^2)$.

(Electro-I-6N1)



6N2. Suppose that the particle in Ex.6N1 is an electron projected with velocity $v_x = 2.0\times10^6$ m/s at the center of the plates. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C , where will the electron strike the upper plate? (Electro-I-6N2)

6N3. An oil drop of 12 excess electron is held stationary under a constant electric field of $2.55\times10^4\,\mathrm{NC^{-1}}$ in Millikan's oil drop experiment. The density of the oil is $1.26\,\mathrm{g\,cm^{-3}}$. Estimate the radius of the drop. Ignore air density. (Electro-I-6N3)

I.7 (A) ELECTRIC DIPOLE

Electric dipole: A pair of equal and opposite electric charges (*q* and –*q*) separated by a small distance constitutes an electric dipole.

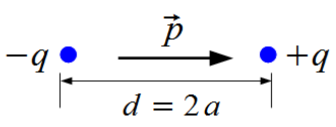


Fig.7.1

The **electric dipole moment** $\vec{p}$ is defined as the product of one of the charges and their separation.

The (electric) dipole moment $\vec{p}$ is a vector quantity. It is directed from negative charge to positive charge:

$$\vec{p} = q\vec{d} \ .$$

Unit of electric dipole moment is coulomb meter (C m).

The dimensions of electric dipole moment are $M^0L^1T^1A^1$.

Examples: Any molecule in which centers of positive and negative charges do not coincide may be treated as a dipole. The molecules such as HCl, H_2O , CO have permanent electric dipoles and are called polar molecules.

I.7(B) ELECTRIC FIELD DUE TO A DIPOLE AT AXIAL AND EQUATORIAL POSITION:

(I) Electric field at an axial point of a dipole.

Consider an electric dipole situated at the point O. The charges $\pm q$ are located at B and A, at distances $\pm a$ from the origin O which is also the center of the dipole.

The point P is on the axis of the dipole, where the net electric field is to be determined (see Fig. 7.2). It is at a distance of *r* from the center O of the dipole.

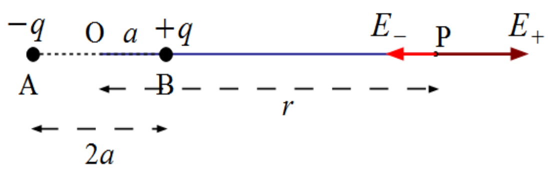


Fig.7.2

The electric fields produced by the $+q$ charge at point P is along BP and is of magnitude

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \quad .$$

The electric fields produced by the $-q$ charge at the point P is along PA and is of magnitude

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \quad .$$

The resultant electric field at P is, therefore,

$$\vec{E} = \vec{E}_+ + \vec{E}_- \quad ,$$

or $$E = E_+ - E_- \quad .$$

The minus sign arises because $\vec{E}_-$ is directed opposite to $\vec{E}_+$.

Thus,

$$E = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right\} ,$$

or $$E = \frac{q}{4\pi\epsilon_0}\left\{\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right\} ,$$

or $$E = \frac{4qar}{4\pi\epsilon_0 \, (r^2-a^2)^2} \quad .$$

Here the magnitude of the dipole moment of the electric dipole is $p=q(2a)$. Therefore the magnitude of the electric field at an axial point of an electric dipole is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{2pr}{(r^2-a^2)^2} \quad .$$

It is directed along the dipole moment vector $\vec{p}$. Therefore in vector form

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}r}{(r^2-a^2)^2} \quad .$$

In the limit $r >> a$, the above expression reduces to

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad .$$

(II) Electric field at an equatorial (bisector) point of a dipole.

Consider an electric dipole situated at the point O. The charges $\pm q$ are located at a distance $\pm a$ from the origin O which is also the center of the dipole.

The point P, where the net electric field is to be determined, is on the bisector of the dipole (see Fig.7.3). The distance of the point P from the center O of the dipole is r.

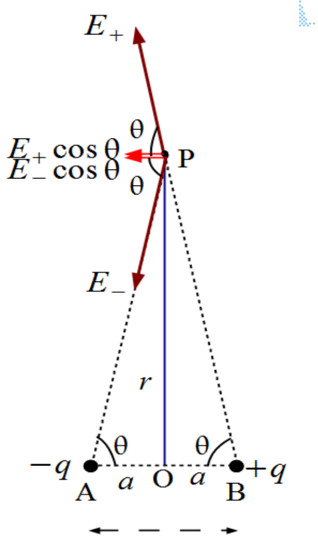


Fig.7.3

The electric field at point P due to $+q$ charge is directed along BP and its magnitude is

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{\{\sqrt{(r^2+a^2)}\}^2}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \; .$$

The electric field at point P due to – q charge is directed along PA and its magnitude is

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{\{\sqrt{(r^2+a^2)}\}^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}$$

The vertical components of these fields, namely, $E_+ \sin\theta$ and $E_- \sin\theta$, cancel each other.

The horizontal components, namely, $E_+ \cos\theta$ and $E_- \cos\theta$, add up to give the resultant field.

Therefore at the equatorial point, the resultant electric field is

$$E = E_+ \cos\theta + E_- \cos\theta \quad ,$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2q\cos\theta}{(r^2+a^2)} \quad ,$$

here

$$\cos\theta = \frac{a}{\sqrt{(r^2+a^2)}} \quad .$$

Therefore, the electric field at the equatorial point is

$$E = \frac{1}{4\pi\epsilon_0}\frac{2qa}{(r^2+a^2)^{3/2}} \quad ,$$

or

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{(r^2+a^2)^{3/2}} \quad .$$

The field is directed opposite to the electric dipole moment $\vec{p}$. Therefore, in the vector form, the field at an equatorial point of a dipole can be written as

$$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{(r^2+a^2)^{3/2}} \quad .$$

In the limit $r >> a$, the above expression reduces to

$$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \quad .$$

(III) Ratio of axial field to equatorial field

The ratio of magnitudes of electric fields at an axial point and at an equatorial point is, (for $r >> a$),

$$\frac{E_{axial}}{E_{equatorial}} = 2 \quad .$$

(IV) Dipole field at arbitrary point

The situation is described in the Fig.7.4. The electric dipole of moment $\vec{p}$ is oriented along the z-axis. We wish to determine the electric field at a point P whose polar coordinates are (r, θ).

We resolve the vector $\vec{p}$ into two directions:
(i) along $\hat{r}$ (the radial direction), and
(ii) along $\hat{\theta}$ (the angular direction).

These component are $p\cos\theta$ and $p\sin\theta$.

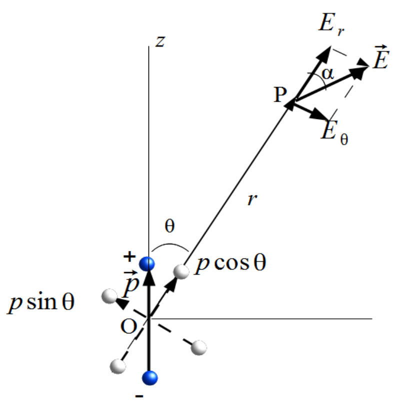


Fig.7.4

The point P is an axial point for $p\cos\theta$ component of the dipole moment. This produces a radial component of electric field. The magnitude of this field is given by

$$E_r = k\frac{2(p\cos\theta)}{r^3}$$

where

$$k = \frac{1}{4\pi\epsilon_0} \quad .$$

The point P is an equatorial point for the $p\sin\theta$ component of the dipole moment. This produces an angular component of electric field. The magnitude of this component is given by

$$E_\theta = k\frac{(p\sin\theta)}{r^3} \quad .$$

The resultant electric field at point P is, then,

$$E = \sqrt{E_r^2 + E_\theta^2} \quad ,$$

or

$$E = \frac{k\,p}{r^3}\sqrt{1+3\cos^2\theta} \quad .$$

The resultant electric field E makes an angle α with the position vector of point P. This angle is such that

$$\tan\alpha = \frac{1}{2}\tan\theta \quad .$$

The field has cylindrical symmetry.

Comments:

(i) Question: Let a point electric dipole is situated at the origin of a Cartesian coordinate system, with dipole moment pointing towards the positive z-axis. Write the Cartesian components of the electric field produced at a point P(x,y,z).

Answer : For an electric dipole pointing in the + z direction, the field components are

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{3xzp}{r^5} \quad , \quad E_y = \frac{1}{4\pi\epsilon_0}\frac{3yzp}{r^5} \quad ,$$

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{(3z^2 - r^2)p}{r^5}$$

(ii) When more that one dipole is present, the net dipole moment is the vector sum of the individual moments.

(iii) For a dipole, when $d \to 0$ and $q \to \infty$ in such a manner that p remains finite, the the dipole is called a point dipole. For a point dipole,

$$\vec{E}_{axial} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad , \quad \vec{E}_{equatorial} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

(iv) The monopole (single charge) field is

$$E \propto \frac{1}{r^2} \text{ , } \textit{a long range field}$$

(v) For a dipole (total charge of system = 0) the field is

$$E \propto \frac{1}{r^3} \text{ , } \textit{a short range field}$$

(vi) For a quadrupole (total charge of system = 0) the field is

$$E \propto \frac{1}{r^4} \text{ , } \textit{a short range field} \quad .$$

i.e., for many charge distributions for which total charge = 0, the nearby electric field may not be zero.

I.7(C) TORQUE ON A DIPOLE IN UNIFORM ELECTRIC FIELD

Let an electric dipole of dipole moment $\vec{p}$ is placed at an angle θ with the electric field $\vec{E}$. The separation between $\pm q$ charges is d. The forces acting on the $+q$ and $-q$ charges, are respectively,

$$\vec{F}_+ = q\vec{E} \text{ , and}$$

$$\vec{F}_- = -q\vec{E} \text{ .}$$

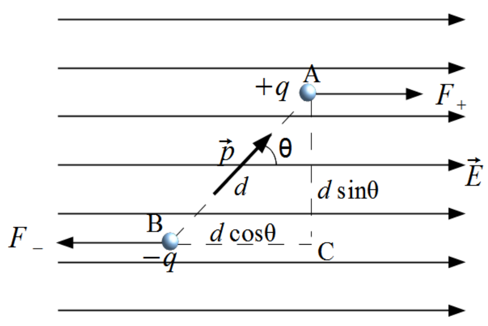


Fig.7.5

These forces are equal in magnitude and opposite in direction. As a result the net force on an electric dipole placed in a uniform electric field is zero.

$$\vec{F} = \vec{F}_+ + \vec{F}_- = 0 \text{ .}$$

The forces acting on the dipole constitute a couple. This couple tries to orient the dipole along the direction of the electric field.

The torque applied by these forces is

$$torque = force \times arm(AC) \, of \, the \, couple$$

or $\tau = (qE) d\sin\theta = (qd) E\sin\theta$,

or $\tau = pE\sin\theta$,

In vector form

$$\vec{\tau} = \vec{p} \times \vec{E} \text{ .}$$

The torque depends on the angle between the dipole moment $\vec{p}$ and the electric field $\vec{E}$.

The torque is maximum when the dipole $\vec{p}$ is perpendicular ($\theta = 90^0$) to the electric field $\vec{E}$.

The torque tends to align the dipole with the field.

The torque is zero when $\vec{p}$ and $\vec{E}$ are parallel.

I.7(D) DIPOLE IN A NON-UNIFORM ELECTRIC FIELD

If electric field is not uniform, then the net force on the electric dipole is non zero.

One situation is illustrated in the Fig.7.6 below.

The dipole moment $\vec{p}$ is pointing in the direction of increasing electric field.

The total force on the $\pm q$ charges of the dipole is

$$F = q(E_+ - E_-) \text{ .}$$

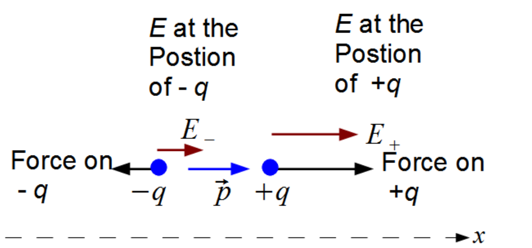


Fig. 7.6

Let the field $\vec{E}$ increases with increase in x and the separation between the charges constituting the dipole is Δx .

Then the total force on the dipole can be written as

$$F_x = q\,\Delta x\,\frac{\Delta E}{\Delta x}\ .$$

In the limit $\Delta x \to 0$, (and since the dipole is oriented along the x axis),

$$F_x = p\,\frac{dE}{dx}\ .$$

This is the expression for the force on a dipole placed in a non-uniform electric field.

In general, for an arbitrary orientation of $\vec{p}$ there exist a net force and a torque on the dipole placed in a non-uniform field. In general, the force and the torque depends on the orientation of $\vec{p}$ with respect to $\vec{E}$.

Conceptual Questions

7C1. Define an electric dipole and electric dipole moment. (Electro-I-7C1)

7C2. Write the units and dimensions of electric dipole moment. Give examples of electric dipoles. (Electro-I-7C2)

7C3. Determine the expression for the electric field at an axial point of an electric dipole of moment $\vec{p}$. (Electro-I-7C3)

7C4. Determine the expression for the electric field at an equatorial point of an electric dipole of moment $\vec{p}$. (Electro-I-7C4)

7C5. What is the ratio of magnitudes of electric field at an axial point and electric field at an equatorial point for a small dipole? Both the points are at a distance of r from the center of the dipole. (Electro-I-7C5)

7C6. Determine the expression for the electric field at an arbitrary point (r, θ) produced by a point electric dipole of dipole of moment $\vec{p}$ situated at the origin and oriented along the z-axis. (Electro-I-7C6)

7C7. Draw a figure qualitatively depicting the electric field of an electric dipole. The dipole is oriented along the z- axis. (Electro-I-7C7)

7C8. An electric dipole of dipole moment ***p*** is placed in a uniform electric field ***E*** making an angle θ. Obtain expressions for the force and the torque on this dipole. When is this torque (i) maximum, (ii) zero? (Electro-I-7C8)

7C9. An electric dipole of dipole moment ***p*** is placed in a non-uniform electric field with dipole moment pointing in the direction of increasing field. What is the expression for the force acting on the dipole? (Electro-I-7C9)

7C10. What is a point electric dipole? What is the ratio of electric field intensity at a distance r on the equatorial line and electric field intensity at a distance r on the axis of the dipole?(Electro-I-7C10)

7C11. A given charge is situated at a certain distance on the axis of a small dipole, and experiences a force F. What force will act on the given charge if its distance along the axis from the dipole is doubled? (Electro-I-7C10)

Numerical Questions

7N1. A dipole consists of two point charges ± 2 nC separated by 4 cm. What is the dipole moment? (Electro-I-7N1)

7N2. The water molecule has a dipole moment

$p = 6.2 \times 10^{-30}$ C m . Find the force on an ion of charge + e at a distance of 0.5 nm: (a) along $\vec{p}$; (b) normal to $\vec{p}$. (use the far field approximation.) (Electro-I-7N2)

7N3. A system has two charges $q_A = 0.25 \mu$ C and $q_B = -0.25 \mu$C located at points A:(0,0, -15 cm) and B:(0,0, +15 cm), respectively. What are the total charge and electric dipole moment of the system? (Electro-I-7N3)

7N4. An electric dipole with dipole moment 4×10^{-9} C m is aligned at $30°$ with the direction of a uniform field of magnitude $5 \times 10^4 \,\text{NC}^{-1}$ .Calculate the magnitude of the torque acting on the dipole? (Electro-I-7N4)

7N5. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive *z*-direction, at the rate of $10^5 \,\text{NC}^{-1}$ per meter. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C m in the negative z-direction? (Electro-I-7N5)

I.8 ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTIONS (PROBLEM SOLVING GUIDE) (FOR IIT-JEE TYPE PREPARATIONS ?)

(I) Line (linear) charge distribution

Let the linear charge density of a line charge distribution is λ (units: C/m). The amount of charge in a line element of length $\Delta r'$ is

$$\Delta q = \lambda \Delta r' \ .$$

The position vector of the line element is $\vec{r}\,'$ (see Fig.8.1). The position vector of a point P is $\vec{r}$. The position vector of the point P with respect to the line element is $\vec{R}$.

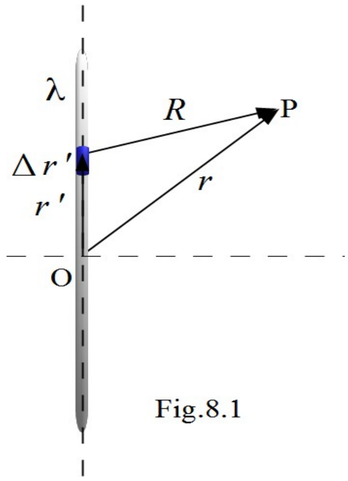


Fig.8.1

The electric field at point P due to this line element is

$$\Delta \vec{E} = \frac{k \Delta q}{R^2} \hat{R}$$

where

$$k = \frac{1}{4 \pi \epsilon_0} \ .$$

Since $\vec{R} = \vec{r} - \vec{r}\,'$ and $\Delta q = \lambda \Delta r'$, the resultant electric field at point P due to the entire linear charge distribution is obtained by integration (many a times difficult to solve)

$$\vec{E} = \int \frac{k \lambda \, dr'}{|\vec{r} - \vec{r}\,'|^3} (\vec{r} - \vec{r}\,') \ .$$

The charge density, if not uniform, is a function of its position *r'*. In that case we have to replace λ by $\lambda(r')$ in the integration.

(II) Surface charge distribution

Let the surface charge density of a surface charge distribution is σ. The amount of charge in an area element ΔS is

$$\Delta q = \sigma \Delta S \quad .$$

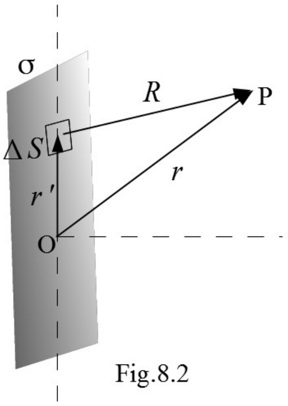


Fig.8.2

The position vector of the area element ΔS is $\vec{r}\,'$ (see Fig.8.2). The position vector of a point P is $\vec{r}$. The position vector of the point P with respect to the area element is $\vec{R}$.

The electric field at point P due to the charges in this area element is

$$\Delta \vec{E} = \frac{k \sigma \Delta S}{R^2} \hat{R} = \frac{k \sigma \Delta S}{R^3} \vec{R} \quad .$$

The total electric field at point P is obtained by evaluating the surface integral

$$\vec{E} = \int_S \frac{k \sigma dS}{|\vec{r} - \vec{r}\,'|} (\vec{r} - \vec{r}\,') \quad .$$

(III) The volume charge distribution

Let the volume charge density of the distribution is ρ. The amount of charge in a volume element $\Delta V'$ of the charge distribution is

$$\Delta q = \rho \Delta V' \quad .$$

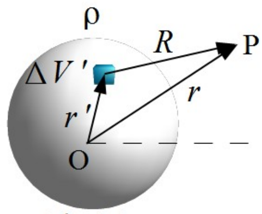


Fig.8.3

The electric field at any point is obtained by proper integration over volume:

$$\vec{E} = \int \frac{k \rho dV'}{|\vec{r} - \vec{r}\,'|^3} (\vec{r} - \vec{r}\,') \quad .$$

Numerical Questions

8N1. A thin insulating rod of length L carries a uniformly distributed charge Q. Find the field strength at a point along its axis at a distance *a* from one end. (Electro-I-8N1)

8N2. A thin insulating semi-infinite rod carries a uniformly distributed charge of linear charge density λ . For a point situated at a perpendicular distance a from one end, determine the electric field components, parallel to the rod and perpendicular to the rod (see Fig.8N2).(Electro-I-8N2)

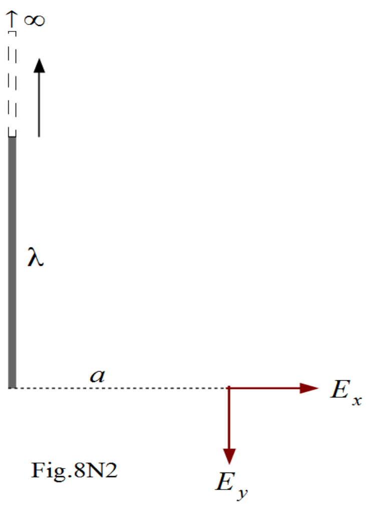


Fig.8N2

SUMMARY FORMULAS

PART – I FORCES, FIELDS AND DIPOLE

Electric Charge

Scalar

Unit: C (coulomb)

$1\,\mathrm{C}=1\,\mathrm{A\cdot s}$

Dim: $\mathrm{M^0L^0T^1A^1}$

$e = 1.602\times10^{-19}\,\mathrm{C}$

Conductor: free electrons

Insulators: No free charges, dipoles, dielectric

Methods of charging:

friction, contact (conduction), induction

Basic properties of charge:

scalar quantity, so additive in nature

conserved

quantized $q = n\,e$

Coulomb's law

$$F=\frac{k\,q_1q_2}{r^2}\quad,\quad F=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$k=1/(4\pi\epsilon_0)$$

$$k=9\times10^9\ \mathrm{N\,m^2C^{-2}}$$

$$\epsilon_0=8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\,m^2}}$$

Dimensions of k are: $\mathrm{M^1L^3T^{-4}A^{-2}}$.

Dimensions of ϵ_0 are: $\mathrm{M^{-1}L^{-3}T^4A^2}$.

$$F=\frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}$$

Dim of permittivity $\epsilon = \mathrm{M^{-1}L^{-3}T^4A^2}$

Relative permittivity or dielectric constant

$$\epsilon_r=\frac{\epsilon}{\epsilon_0}$$

$$F_{medium}=\frac{F_{vacuum}}{\epsilon_r}$$

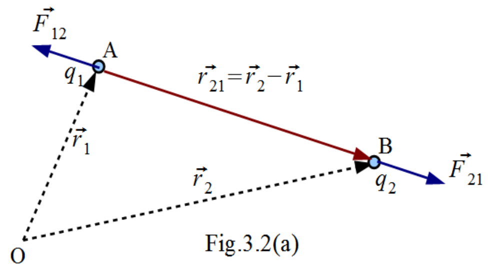


Fig.3.2(a)

$$\vec F_{12}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat r_{12}=-\vec F_{21}$$

Ratio of electric force to gravitational force

$$\frac{F_E}{F_G}=\frac{k\,q_1q_2}{G\,m_1m_2}$$

For p and e

$$\frac{F_E}{F_G}=2.4\times10^{39}$$

For p and p

$$\frac{F_E}{F_G}=1.2\times10^{36}$$

Superposition

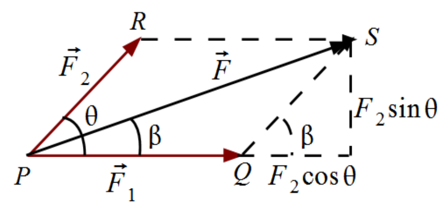


Fig.4.2

$$F=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$$

$$\tan\beta=\frac{F_2\sin\theta}{F_1+F_2\cos\theta}$$

Line (linear) charge density

$$\lambda=\frac{Q}{L}$$

$$\lambda = \lim_{\Delta l \to 0} \left(\frac{\Delta Q}{\Delta l} \right)$$

unit: $\mathrm{C\,m^{-1}}$

Surface charge density

$$\sigma = \frac{Q}{A}$$

$$\sigma = \lim_{\Delta A \to 0} \left(\frac{\Delta Q}{\Delta A} \right)$$

unit: $\mathrm{C\,m^{-2}}$

Volume charge density

$$\rho = \frac{Q}{V}$$

$$\rho = \lim_{\Delta V \to 0} \left(\frac{\Delta Q}{\Delta V} \right)$$

unit: $\mathrm{C\,m^{-3}}$

Electric field

Vector quantity

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

unit: $(\mathrm{N\,C^{-1}})$ or $\mathrm{V\,m^{-1}}$

dim: $\mathrm{M^1 L^1 T^{-3} A^{-1}}$

Force on a charge placed in an electric field

$$\vec{F} = q\vec{E}$$

Electric field due to a point charge

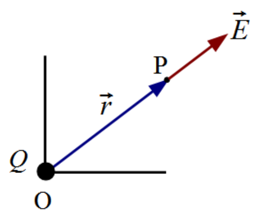


Fig.5.2(b)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

Principle of superposition

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

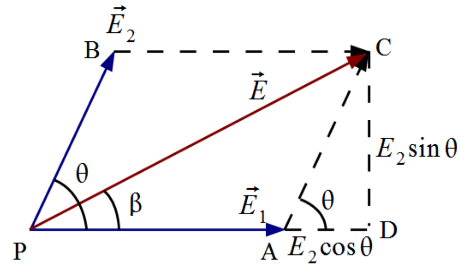


Fig.5.5

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

$$\tan\beta = \frac{E_2 \sin\theta}{E_1 + E_2\cos\theta}$$

Ring charge- electric field at an axial point

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$$

Motion of charged particle in an electric field

(case - I)

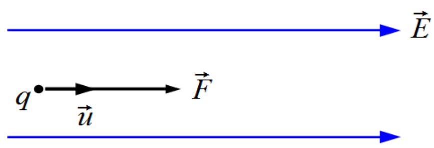


Fig.6.1

$$\vec{F} = q\vec{E}$$

$$\vec{a} = \frac{q\vec{E}}{m}$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + (1/2)\vec{a}t^2$$

(case – II)

$F_x = 0, a_x = 0$

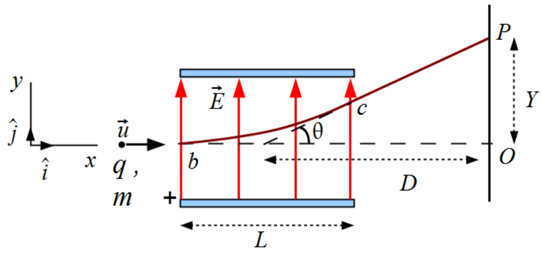


Fig.6.2

$F_y = qE, \quad a_y = qE/m$

$V_x = u$

$V_y = \frac{qEt}{m}$

$x = ut$

$y = \frac{qEt^2}{m}$

$y = \frac{1}{2}\frac{qE}{mu^2}x^2$

$t' = \frac{L}{u}$

$y' = \frac{qEL^2}{2mu^2}$

$V_y = \frac{qEL}{mu}$

$V = \sqrt{u^2 + (\frac{qEL}{mu})^2}$

$\tan\theta = \frac{V_y}{V_x} = \frac{qEL}{mu^2}$

$Y = D\tan\theta = \frac{qELD}{mu^2}$

$= \frac{qELD}{2K}$.

Millikan Oil Drop Experiment

$qE = m_{eff}\, g$

$m_{eff} = (4/3)\pi r^3(\rho - \rho_A)$

$qE = \frac{4}{3}\pi r^3(\rho - \rho_A)g$

$6\pi\eta r v_T = m_{eff}\, g$

$r = \sqrt{\frac{9}{2}\frac{\eta v_T}{(\rho - \rho_A)g}}$

$q = \frac{18\pi}{E}\sqrt{\frac{\eta^3 v_T^3}{2(\rho - \rho_A)g}}$

Electric dipole

$\vec{p} = q\vec{d}$

Dim of p : $M^0L^1T^1A^1$

Electric field at an axial point of a dipole

$E = \frac{1}{4\pi\epsilon_0}\frac{2pr}{(r^2 - a^2)^2}$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}r}{(r^2 - a^2)^2}$

For $r >> a$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$

Electric field at an equatorial point of a dipole

$E = \frac{1}{4\pi\epsilon_0}\frac{p}{(r^2 + a^2)^{3/2}}$

$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{(r^2 + a^2)^{3/2}}$

For $r >> a$

$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$

$\frac{E_{axial}}{E_{equatorial}} = 2$

At arbitrary point (r, θ)

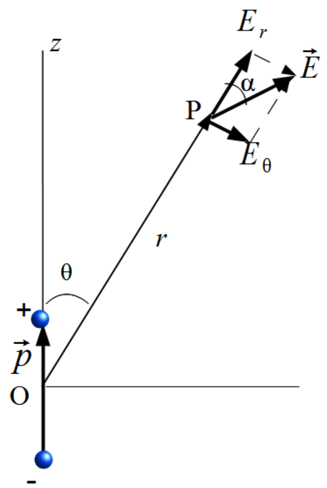


Fig.7.4a

$$E_r = k\frac{2(p\cos\theta)}{r^3}$$

$$E_\theta = k\frac{(p\sin\theta)}{r^3}$$

$$E = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{1}{2}\tan\theta$$

Torque

$$\tau = pE\sin\theta$$

$$\vec{\tau} = \vec{p}\times\vec{E}$$

Force on a dipole placed in a non-uniform electric field (special case)

$$F_x = p\frac{dE}{dx} \quad .$$

ANSWERS OF NUMERICAL QUESTIONS

2N1. 19.8 years

2N2. (a) 2×10^{12} , from wool to polythene,

(b) yes, but a negligible amount

2N3. 6.2×10^{18}electrons , 5.6×10^{-12}kg

2N4. 2.16×10^{-11}

3N1. 12×10^{-3} N , repulsion

3N2. (a) 12 cm,

(b) 0.8 N attractive

3N3. $a_e = 2.5\times10^{22}\,\text{m/s}^2$ and

$a_p = 1.4\times10^{19}\,\text{m/s}^2$

3N4. 9.6×10^{6}C

3N5. $F_G = 7.4\times10^{-10}$ N , and $F_E = 6\times10^{-3}$ N

3N6. 4F / 3

3N7. $q = Q/2$

3N8. (a) 1.5×10^{-2}N , (b) 0.24 N

3N9. 5.7×10^{-3} N

3N10. (a) 15 μC, 5 μC , (b) 35 μC, 15 μC

3N11. (a) 3.6×10^{-4} N attractive,

(b) 2.0×10^{-4} N repulsive

3N12. 0.24 μC

3N13. $q_1 = q_2 = Q/2$

4N1. $F = \sqrt{3}kQ^2/a^2$

4N2. Zero

4N3. $F = (1+2\sqrt{2})\frac{kq^2}{2a^2}$

4N4. distance from q_1, $x = R/(1+\sqrt{q_2/q_1})$,

and distance from q_2, $d = R/(1+\sqrt{q_1/q_2})$.

4N5. See solution

4N6. Zero

4N7. At a distance $d = R/(\sqrt{|q_1|/|q_2|}-1)$ from charge $-q_2$, and at a distance (R + d) from the charge q_1 .

4N8. Force on charge q at A is $q^2/(4\pi\epsilon_0 l^2)$, the force on charge q at B is $q^2/(4\pi\epsilon_0 l^2)$, and the force on charge $-q$ at C is $\sqrt{3}q^2/(4\pi\epsilon_0 l^2)$.

4N9. $8.1\times10^{-3} N$.

5N1. (a) $25\hat{i}$ N/C , (b) Nothing, the electric field does not depend on the test charge.

5N2. At a distance x from the charge Q_1 and (R – x) from the charge Q_2 on the line joining the two charges, where $x=R/(1+\sqrt{Q_2/Q_1})$.

5N3. At a distance x from the smaller magnitude charge $-Q_2$, and distance (R + x) from Q_1 on the line joining the two charges, where $x=R/(\sqrt{|Q_1|/|-Q_2|}-1)$.

5N4. $E_A=7.2\times10^4$ N/C directed toward the right, $E_B=3.2\times10^4$ N/C directed toward the left, $E_C=9\times10^3$ N/C .

5N5.(a) 5.4×10^6 N/C along OB,
(b) 8.1×10^{-3} N along OA

6N1. See solution

6N2. 1.6 cm

6N3. 9.8×10^{-4} mm

7N1. 8×10^{-11} C m

7N2. (a) 1.43×10^{-10} N , (b) 0.71×10^{-10} N

7N3. The total charge is zero. Dipole moment is 7.5×10^{-8} C m along z-axis.

7N4. 10^{-4} N m

7N5. The force is 0.01 N in the negative z-direction, that is the direction of decreasing field (this is also the direction of the decreasing potential energy of the dipole. The torque is zero.)

8N1. $E=\frac{kQ}{a(a+L)}$

8N2. $E_x=E_y=\frac{k\lambda}{a}$

For solutions of conceptual questions and numerical questions, follow the following steps:

1. LOGIN with your ID and password.
2. Click – view ANSWER DETAIL
3. Type – Question ID.
4. Click – VIEW ANSWER

LOGOUT before you leave.

For MCQ TEST

LOGIN

Select a TEST

Start the TEST

After the TEST is over, view your score (Test report). You can also view solutions of the MCQ questions.