(C) NUMERICAL QUESTIONS

N1. The gravitational constant G appears in the following formula

$$F = \frac{G m_1 m_2}{r^2} \quad ,$$

where F is the force of gravitational attraction between two masses m_1 and m_2 separated by a distance r. Obtain the derived SI units for G.

N2. Calculate the angle of (a) 1° (degree) (b) 1′ (minute of arc or arcmin) and (c) 1″ (second of arc or arc second) in radians. Use $360^0 = 2\pi$ rad, 1°=60′ , 1′ = 60″, $\pi = 3.141$.

N3. Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to .....m^3 .

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ......$(mm)^2$.

(c) A vehicle moving with a speed of 18 km h^{-1} covers.........m in 1 s .

(d) The relative density of lead is 11.3. Its density is ......g cm^{-3} or …….kg m^{-3}.

N4. Fill in the blanks by suitable conversion of units:

(a) 1 kg m^2 s^{-2} = ...g cm^2 s^{-2} .

(b) 1 m = …..... ly (light year).

(c) 3.0 m s^{-2} = ….... km h^{-2} .

(d) G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$

= ….... $(cm)^3$ s^{-2} g^{-1}.

N5. A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 $\alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

N6. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

N7. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

N8. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

N9. A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO (see Fig. N9); but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40^0 (θ is known as 'parallax') estimate the distance of the tower C from his original position A.

N10. The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is $1^\circ\ 54'$. Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth.

N11. The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11}$ m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (one second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (one second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres? (Comment: In this question taken from NCERT book, the term "parallax" is referred to the **semi-angle of inclination** between two sight-lines to the object.)

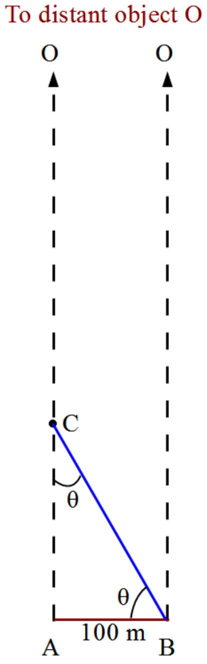

Fig. N9

N12. The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

N13. The Sun's angular diameter is measured to be 1920″. The distance *D* of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ?

N14. If the size of a nucleus (in the range of 10^{-15} to 10^{-14} m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.

N15. When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of the Jupiter.

N16. A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

N17. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = $1450\ m\ s^{-1}$).

N18. The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

N19. It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. The Sun's angular diameter is measured to be 1920" . The distances of the Sun and the moon from the Earth are, 1.496×10^{11} m and 3.84×10^{8}m , respectively. From these facts determine the approximate diameter of the moon.

N20. (a) The earth-moon distance is about 60 earth radius. What will be the diameter of the earth (approximately in degrees) as seen from the moon?

(b) Moon is seen to be of (½)°diameter from the earth. What must be the relative size compared to the earth?

(c) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters.

N21. (a) How many astronomical units (A.U.) make 1 parsec?

(b) Consider a sun-like star at a distance of 2 parsecs. When it is seen through a telescope with 100 magnification, what should be the angular size of the star? Sun appears to be (1/2)° from the earth. Due to atmospheric fluctuations, eye can't resolve objects smaller than 1 arc minute.

(c) Mars has approximately half of the earth's diameter. When it is closest to the earth it is at about 1/2 A.U. from the earth. Calculate what size it will appear when seen through the same telescope.

(Comment : This is to illustrate why a telescope can magnify planets but not stars.)

N22. In an experiment to estimate the size of a molecule of oleic acid 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now 1 drop of this diluted solution is placed on water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, lycopodium powder is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and area of the film we can calculate the thickness of the film which will give us the size of oleic acid molecule. Read the passage carefully and answer the following questions:

(a) Why do we dissolve oleic acid in alcohol?

(b) What is the role of lycopodium powder?

(c) What would be the volume of oleic acid in each mL of solution prepared?

(d) How will you calculate the volume of n drops of this solution of oleic acid?

(e) What will be the volume of oleic acid in one drop of this solution?

N23. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. Write the density of the material of the body in correct significant figures.

N24. The length and breadth of a rectangular sheet are 16.2 cm and 10.1cm, respectively. What is the area of the sheet in appropriate significant figures and error in it?

N25. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

N26. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

N27. Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

N28. 5.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view.

N29. Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are :

	Clock 1	**Clock 2**
Monday	12:00:05	10:15:06
Tuesday	12:01:15	10:14:59
Wednesday	11:59:08	10:15:18
Thursday	12:01:50	10:15:07
Friday	11:59:15	10:14:53
Saturday	12:01:30	10:15:24
Sunday	12:01:19	10:15:11

If you are doing an experiment that requires precision **time interval** measurements, which of the two clocks will you prefer?

N30. Which of the following measurements is most precise?
(a) 5.00 mm ; (b) 5.00 cm ; (c) 5.00 m ; (d) 5.00 km.

N31. The mean length of an object is 5 cm. Which of the following measurements is most accurate?
(a) 4.9 cm; (b) 4.805 cm; (c) 5.25 cm; (d) 5.4 cm.

N32. The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in the measurement of distance.

N33. Time for 20 oscillations of a pendulum is measured as t_1= 39.6 s; t_2= 39.9 s; t_3= 39.5 s. (a) What is the precision in the measurements? (b) What is the accuracy of the measurements?

N34. We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s. Calculate (a) the mean absolute error, (b) relative error , (c) percentage error.

N35. The temperatures of two bodies measured by a thermometer are t_1 = 20 ºC ± 0.5 ºC and t_2 = 50 ºC ± 0.5 ºC. Calculate the temperature difference and the error their in.

N36. The resistance $R = V/I$ where V = (100 ± 5)V and I = (10 ± 0.2)A. Find the percentage error in R.

N37. Two resistors of resistances R_1 = 100 ±3 ohm and R_2 = 200 ± 4 ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$, and for (b) the relations

$$\frac{1}{R'}=\frac{1}{R_1}+\frac{1}{R_2} \quad \text{, and} \quad \frac{\Delta R'}{R'^2}=\frac{\Delta R_1}{R_1^2}+\frac{\Delta R_2}{R_2^2} \; .$$

N38. Find the relative error in Z, if

$$Z=\frac{A^4 B^{1/3}}{C D^{3/2}} \quad .$$

N39. The period of oscillation of a simple pendulum is $T=2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g?

N40. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?

N41. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?

N42. It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?

N43. Measure of two quantities along with the precision of respective measuring instrument is

$$A = 2.5 \text{ m s}^{-1} \pm 0.5 \text{ m s}^{-1}$$

$$B = 0.10 \text{ s} \pm 0.01 \text{ s}$$

What will be the value of product AB?

N44. You measure two quantities as A = 1.0 m ± 0.2 m, B = 2.0 m ± 0.2 m. How should one report correct value for $\sqrt{AB}$?

N45. A physical quantity P is related to four observables a, b, c and d as follows :

$$P=\frac{a^3 b^2}{\sqrt{c d}}$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. (I) What is the percentage error in the quantity P ? (II) If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result ?

N46. A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion :

(a) $y = a \sin 2\pi t / T$

(b) $y = a \sin vt$

(c) $y = (a/T) \sin (t / a)$

(d) $y = (a\sqrt{2}) (\sin 2\pi t / T + \cos 2\pi t / T)$

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

N47. A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :

$$m=\frac{m_0}{(1-v^2)^{1/2}}$$

Guess where to put the missing c.

N48. Let us consider an equation

$$\frac{1}{2}mv^2 = mgh$$

where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

N49. The SI unit of energy is J = kg m^2 s^{-2}; that of speed v is m s^{-1} and of acceleration a is m s^{-2}. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body) :

(a) $K = m^2 v^3$

(b) $K = (1/2)mv^2$

(c) $K = ma$

(d) $K = (3/16)mv^2$

(e) $K = (1/2)mv^2 + ma$

N50. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

N51. Einstein's mass - energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as $E = mc^2$, where

$c = 2.9979 \times 10^8$ m/s

is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where

1 MeV= 1.6022×10^{-13} J ;

the masses are measured in unified atomic mass unit (u) where

1 $u = 1.6605 \times 10^{-27}$ kg.

(a) Show that the energy equivalent of 1u is 931.5MeV.

(b) A student writes the relation as 1 u = 931.5 MeV. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

N52. A physical quantity X is related to four measurable quantities a, b, c and d as follows:

$$X = a^2 b^3 c^{5/2} d^{-2} \quad .$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 1%, respectively. (a) What is the percentage error in quantity X ? (b) If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

N53. In the expression

$$P = E L^2 m^{-5} G^{-2}$$

E, m, L and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

N54. If velocity of light c, Planck's constant h and gravitational constant G are taken as fundamental quantities then express mass, length and time in terms of dimensions of these quantities.

N55. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional analysis, that

$$T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$$

where k is a dimensionless constant and g is acceleration due to gravity.

N56. The volume of a liquid flowing out per second ($\tilde{V}$)of a pipe of length l and radius r is written by a student as

$$\tilde{V} = \frac{\pi}{8}\frac{P r^4}{\eta l} ,$$

where P is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $ML^{-1}T^{-1}$. Check whether the equation is dimensionally correct.

N57. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data : mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^{8} m.

N58. A man walking briskly in rain with speed v must slant his umbrella forward making
an angle θ with the vertical. A student derives the following relation between θ and v : tanθ = v and checks that the relation has a correct limit: as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

N59. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Take - Atomic mass of sodium M =23, and Avogadro number $N_A \approx 6\times10^{23}\,\text{mol}^{-1}$). Compare it with the density of sodium in its crystalline phase : $970\ \text{kg m}^{-3}$. Are the two densities of the same order of magnitude? If so, why?

N60. The unit of length convenient on the nuclear scale is a fermi (femtometer) : 1f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation :

$$r = r_0 A^{1/3} ,$$

where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in question N59.