(D) Answers of Conceptual Questions

1. $m^3 kg^{-1} s^{-2}$
2.(a) 1.745×10^{-2} rad , (b) 2.908×10^{-4} rad , (c) 4.847×10^{-6} rad
3.(a) 10^{-6} , (b) $1.51\times10^4 (mm)^2$, (c) 5 m , (d) 11.3 g/cm^3 or 1.13×10^4 kg/m^3
4.(a) 10^7 , (b) 10^{-6} , (c) 3.9×10^4 , (d) 6.67×10^{-8}
6. 500
7. $V\approx3\times10^{-7}\ m^3$
8. $r\approx10^4$
9. 119 m
10. 3.84×10^8 m
11. $\approx3\times10^{16}$ m)
12. 1.32 pc, 1.52″
13. 1.39×10^9 m
14. ≈1 m
15. 1.429×10^5 km
16. 3.84×10^8 m
17. 55.8 km
18. 2.8×10^{22} km
19. 3580 km
20. (a) 1.9°, (b) $D_{Earth}\approx4D_{moon}$, (c) $D_{Sun}/D_{Earth}=100$
21. (a) 1 parsec $\approx2\times10^5$ A.U. , (b) 15×10^{-3} min arc , (c) 30′
23. 1.7 g/cm^3
24. 164 ± 3 cm^2
25. 8.72 m^2 , 0.0855 m^3
26. (a) 2.3 kg, (b) 0.02 g
27. 311.3 m^2 , 373.7 m^3
28. 4.8 g/cm^3
29. Clock 2
30. (a)
31. (a)
32. 0.01 mm
33. (a) 0.005 s , (b) maximum error = 0.012 s
34. (a) 0.11 s, (b) 0.04, (c) 4%
35. $30^\circ C\pm1^\circ C$
36. 7%
37. (a) (300 ±7) ohm, (b) (66.7 ± 1.8) ohm
38. $\frac{\Delta Z}{Z}=4\frac{\Delta A}{A}+\frac{1}{3}\frac{\Delta B}{B}+\frac{\Delta C}{C}+\frac{3}{2}\frac{\Delta D}{D}$
39. 3%
40. 0.035 mm
41. 94.1
42. 1 part in 10^{11} to 10^{12}
43. (0.25 ± 0.08)m
44. (1.4 ± 0.2) m
45. (I) 13% , (II) 3.8
46. (b) and (c) are wrong
47. $m=m_0\left(1-(v^2/c^2)\right)^{-1/2}$
48. Correct
49. (a), (c) and (e) ruled out, (b) and (d) allowed
50. $T=k\sqrt{l/g}$
51. (b) $1\,u=931.5\,MeV/c^2$
52. (a) $\frac{\Delta X}{X}=12.5\%$, (b) $X=2.8\pm0.3$
53. dimensionless
54. (1) $\sqrt{\frac{ch}{G}}$, (2) $\sqrt{\frac{hG}{c^3}}$, (3) $\sqrt{\frac{hG}{c^5}}$
56. valid
58. $\tan\theta=v/v_{rain}$
59. $\rho_{Na}=4.6\times10^2\,kg/m^3$.
60. $\rho_{nucleus}=2.3\times10^{17}$ kg/m^3

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Solutions:

N1. From the given formula, the gravitational constant *G* may be written as

$$G = \frac{F r^2}{m_1 m_2}$$

Substituting SI units for force, distance and mass, we get the following SI units for *G*:

$$\text{SI unit of } G = \frac{(\text{kg m s}^{-2})(\text{m}^2)}{\text{kg}^2} \quad ,$$

or $$\text{SI unit of } G = \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \quad .$$

N2. (a) Since $2\pi(rad) = 360^\circ$,

$$1^\circ = \frac{\pi}{180} = \frac{3.141}{180} = 1.745\times10^{-2}\,\text{rad} \quad .$$

(b) $$1' = \frac{1^\circ}{60} = \frac{1.745\times10^{-2}}{60} = 2.908\times10^{-4}\,\text{rad} = 2.91\times10^{-4}\,\text{rad} \quad .$$

(c) $$1'' = \frac{1'}{60} = \frac{2.908\times10^{-4}}{60} = 4.847\times10^{-6} = 4.85\times10^{-6}\,\text{rad} \quad .$$

N3. (a) 10^{-6} .

(b)Surface area = $2\pi r(r+l)$,

$= 2\times3.14\times20\times120\,(\text{mm})^2$,

$= 1.51\times10^4\,(\text{mm})^2$.

(c) distance = speed × time

$$= 18\left(\frac{\text{km}}{\text{h}}\right)\times1\,\text{s} = 18\times\left(\frac{1000\,\text{m}}{3600\,\text{s}}\right)\times1\,\text{s} \quad ,$$

$= 5\,\text{m}$.

(d) Density of water = $1\,\text{g/cm}^3$. Therefore,

Density of lead =(relative density)×(density of water)

$= 11.3\times1 \quad \text{g/cm}^3$

$= 11.3 \quad \text{g/cm}^3$.

or Density lead = $$11.3\times\left(\frac{10^{-3}\,\text{kg}}{10^{-6}\,\text{m}^3}\right) = 1.13\times10^4\frac{\text{kg}}{\text{m}^3} \quad .$$

N4. (a) $$1\,\text{kg m}^2\text{s}^{-2} = (1000\,\text{g})(100\,\text{cm})^2\text{s}^{-2} = 10^7\,\text{g cm}^2\text{s}^{-2} \quad .$$

(b) 1 ly = distance travelled by light (in vacuum) in one year. Therefore,

$$1\,\text{ly} = 3\times10^8\,\text{m s}^{-1}\times(365\times24\times3600\,\text{s}) \quad ,$$

or $$= 9.46\times10^{-8}\,\text{m} \quad .$$

Therefore, $$1\,\text{m} = \frac{1}{(9.46\times10^{-8})}\,\text{ly} \quad ,$$

or $$1\,\text{m} = 1.06\times10^{-6}\,\text{ly} = 10^{-6}\,\text{ly} \quad .$$

(c) $$3\,\text{m s}^{-2} = 3(10^{-3}\,\text{km})\left(\frac{1}{3600}\right)^{-2}\text{h}^{-2} \quad ,$$

or $$= \frac{3}{1000}\times3600^2\,\text{km h}^{-2} \quad ,$$

or $$= 38880\,\text{km h}^{-2} \quad ,$$

or $$= 3.9\times10^4\,\text{km h}^{-2} \quad .$$

(d) $G = 6.67\times10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$,

and $1\,\text{N} = 1\,\text{kg m s}^{-2}$. Therefore,

$$G = 6.67\times10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \quad ,$$

or $$= 6.67\times10^{-11}\times(100\,\text{cm})^3\times(10^3\,\text{kg})^{-1}\times\text{s}^{-2} \quad ,$$

or $$= 6.67\times10^{-11}\times10^6\times\left(\frac{1}{10^3}\right)\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2} \quad ,$$

or $$= 6.67\times10^{-8}\,\text{cm}^3\,\text{g}^{-1}\,\text{s}^{-2} \quad .$$

N5. Let in the new system of units, the length, mass and time units are: m_N , kg_N and s_N . It is given that

$$1\,\text{m}_N = \beta\,\text{m} \quad ,$$
$$1\,\text{kg}_N = \alpha\,\text{kg} \quad ,$$
$$1\,\text{s}_N = \gamma\,\text{s} \quad .$$

Therefore,

$$1\,\text{J}_N = 1\,\text{kg}_N\cdot\text{m}_N^2\cdot\text{s}_N^{-2} \quad ,$$

or $$= (\alpha\,\text{kg})\cdot(\beta^2\,\text{m}^2)\cdot(\gamma^{-2}\,\text{s}^{-2}) \quad ,$$

or $$1\,\text{J}_N = (\alpha\beta^2\gamma^{-2})\,\text{kg m}^2\text{s}^{-2} \quad ,$$

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or $1\,J_N = (\alpha\beta^2\gamma^{-2})\ J$.

Therefore,

$$1\,\text{cal} = 4.2\ J = \frac{4.2}{(\alpha\beta^2\gamma^{-2})}\ J_N \quad ,$$

or $1\,\text{cal} = 4.2\,\alpha^{-1}\beta^{-2}\gamma^2\ J_N$.

N6. In SI units, the speed of light is

$$c = 3\times10^8\ \text{m/s} \quad . \qquad (1)$$

Let the new unit of length is m_N . It is given that

$$c = 1\ m_N/s \quad . \qquad (2)$$

Comparing (1) and (2), we get

$$1\ m_N = 3\times10^8\ \text{m} \quad . \qquad (3)$$

The distance between the Sun and the Earth is

$$d = ct = 3\times10^8(\text{m/s})\times500(\text{s}) = 15\times10^{10}\ \text{m} \quad ,$$

or $d = 15\times10^{10}\times\left(\frac{1}{3\times10^8}\right) m_N = 500\ m_N$.

N7. Avogadro constant is

$$N_A = 6.022\times10^{23}\,\text{mol}^{-1} \quad .$$

The volume of one hydrogen atom is

$$V_1 = \frac{4\pi r^3}{3} = \frac{4\times3.14\times(0.5\times10^{-10})^3}{3}\ \text{m}^3 \quad ,$$

or $V_1 = 5.23\times10^{-31}\ \text{m}^3$.

Therefore, the volume of one mole of hydrogen atoms is

$$V = N_A V_1 = 6.02\times10^{23}\times5.23\times10^{-31}\ \text{m}^3 \quad ,$$

or $V = 3.15\times10^{-7}\ \text{m}^3 \approx 3\times10^{-7}\ \text{m}^3$.

N8. The size of hydrogen molecule is

$$V_1 = \frac{4\pi r^3}{3} = \frac{4\times3.14\times(1\times10^{-10})^3}{3}\ \text{m}^3 \quad ,$$

$$V_1 = 4.19\times10^{-30}\ \text{m}^3 \quad .$$

Number of molecules in one mole of hydrogen (gas) is

$$N = N_A = 6.022\times10^{23} \quad .$$

Therefore, volume of molecules of hydrogen in one mole of gas is

$$V = N\,V_1 = 6.02\times10^{23}\times4.29\times10^{-30}\ \text{m}^3 \quad ,$$

or $V = 2.58\times10^{-6}$.

The volume of one mole of gas

$$V_{gas} = 22.4\,\text{L} = 22.4\times10^{-3}\ \text{m}^3 \quad .$$

Therefore, the ratio molar volume to volume of molecules is

$$r = \frac{V_{gas}}{V} = \frac{22.4\times10^{-3}}{2.58\times10^{-6}} = 8.68\times10^3 \approx 10^4 \quad .$$

This ratio is large because intermolecular separation in a gas is much larger than the size of a molecule.

N9. The parallax angle is 40° . From ΔABC,

AC = AB / tanθ

= 100 / tan40° metre

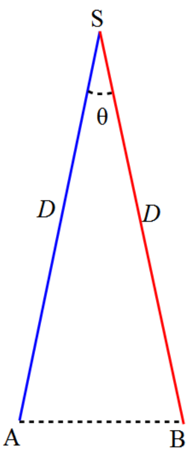
= 100 / 0.8391

= 119.17

= 119 m

N10. According to parallax method

$$D = \frac{b}{\theta} \quad . \qquad (1)$$



Given, $b = 1.276\times10^7\ \text{m}$,

and $\theta = 1^\circ 54' = 114'$.

Converting the degree into radians, we get

$$\theta = \frac{\pi}{(180\times60)}\times114 \quad ,$$

or $\theta = \frac{3.141\times114}{180\times60}$,

or $\theta = 3.32\times10^{-2}\,\text{rad}$.

Substituting values of b and θ in Eq.(1), we get,

$$D = \frac{1.276\times10^7}{3.32\times10^{-2}}\ \text{m} \quad ,$$

$$D = 3.84\times10^8\ \text{m} \quad .$$

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N11. From the given information, the distance parsec is

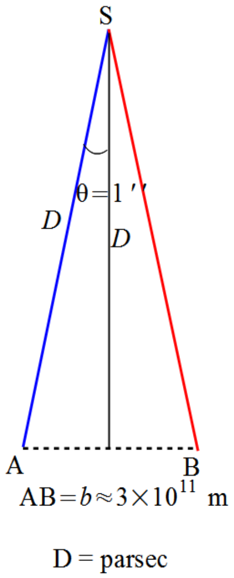
$$D=\frac{(b/2)}{\theta}\ ,$$

where $b\approx 3\times 10^{11}\ \text{m}$, and

$$\theta=1''=\frac{3.141}{180\times 3600}\ .$$
$$=4.85\times 10^{-6}\ \text{rad}$$

Therefore,

$$D\approx\frac{1.5\times 10^{11}}{4.85\times 10^{-6}}\ \text{m}\ .$$
$$\approx 3\times 10^{16}\ \text{m}$$



Comment:

(i) Astronomical unit is approximately equal to the semi-major axis of the Earth's orbit.

(ii) astronomical unit is (symbol *A* or *au*)

$$1\,A=1.49597..\times 10^{11}\ \text{m}\ .$$

(iii) One parsec (pc) is the distance at which 1*A* subtends 1 sec arc.

$$1\ \text{pc}=\frac{1\,A}{1''}=\frac{1\,A}{(\pi/648000)}=\frac{1\,A}{4.85\times 10^{-6}}\ .$$
$$=3.08567..\times 10^{16}\,\text{m}=3.262..\text{ly}$$

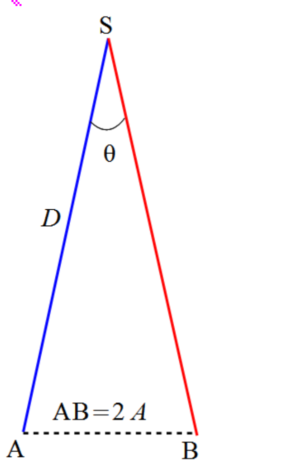
N12. Distance of the star *D* = 4.29 ly. Therefore, in pc unit, the distance of the star is

$$D=\frac{4.29}{3.262}=1.32\ \text{pc}\ .$$

The parallax is

$$\theta=\frac{2\,A}{1.32\ \text{pc}}\ ,$$
$$=\frac{2\,A}{1.32\,A/1''}$$

or $\theta=\frac{2}{1.32}=1.52''$.



A = *au* = astronomical unit

Fig.N12

N13. The diameter of the Sun is

$$d=D\alpha\ .$$

Substituting values, we get

$$d=1.496\times 10^{11}\times 1920\times 4.85\times 10^{-6}$$

or $=\ 1.39\times 10^{9}\ \text{m}$.

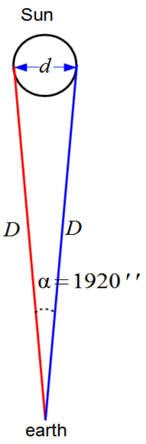


Fig.N13

N14. In the question we are scaling up the size by a factor of

$$\frac{10^{-5}}{10^{-15}}=10^{10}\ .$$

The size of an atom is $\approx 10^{-10}\,\text{m}$. Therefore, it will be scaled up to a size $\approx 10^{-10}\times 10^{10}\approx 1\ \text{m}$.

Comment: A nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

N15. The diameter of the jupitar is

$$d=D\alpha\ .$$

Substituting values, we get

$$d=824.7\times 10^{9}\times 35.72\times 4.85\times 10^{-6}\ \text{m}$$

or $=\ 1.429\times 10^{8}\ \text{m}=1.429\times 10^{5}\ \text{km}$.

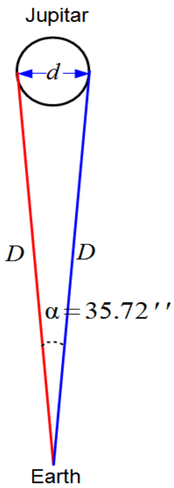


Fig.N15

N16. If *r* is the radius of the lunar orbit around the Earth, the distance travelled by the LASER beam in going to moon and coming back to Earth is 2*r*. If c is the speed of light, and t is the time taken in covering distance 2r, then

$$2r=ct=3\times 10^{8}\left(\frac{\text{m}}{\text{s}}\right)\times 2.56(\text{s})\ .$$
$$=7.68\times 10^{8}\ \text{m}$$

Therefore, $r=\frac{7.68\times 10^{8}}{2}=3.84\times 10^{8}\ \text{m}$.

N17. If *d* is the distance of the enemy submarine, the speed of sound in water is *v* and the time-interval for

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the signal to come back to the detector is t, then

$$2d = vt \quad .$$

It is given that $v = 1450$ m s^{-1} , and $t = 77.0$ s. Therefore,

$$2d = 1450\left(\frac{\text{m}}{\text{s}}\right)\times 77(\text{s}) = 111650 \text{ m} \quad .$$

Thus,

$$d = \frac{111650}{2} = 55825 \text{ m} = 55.8 \text{ km} \quad .$$

N18. Let the distance of the quasar is d , speed of light is c and time taken is t (all in SI units). Then,

$$d = ct \quad .$$

Substituting values, we get

$$\begin{aligned} d &= 3.0\times 10^{8}(\text{m/s})\times 3.0\times 10^{9}\times 365\times 24\times 3600(\text{s}) \\ &= 2.84\times 10^{25} \text{ m} \\ &= 2.8\times 10^{22} \text{ km} \end{aligned} \quad .$$

N19. The situation is illustrated in Fig.N19. The diameter of the moon is

$$d_1 = D_1 \times \beta \quad .$$

Here, the distance of the moon from the Earth is $D_1 = 3.84\times 10^{8}$ m and $\beta = 1920''$. Therefore,

$$\begin{aligned} d_1 &= 3.84\times 10^{8}\times 1920\times 4.85\times 10^{-6} \\ &= 3.58\times 10^{6} \text{ m} \\ &= 3580 \text{ km} \end{aligned} \quad .$$

Fig. N19

N20. The situation is illustrated in Fig.N20a. The angular diameter of the Earth as seen from the moon is

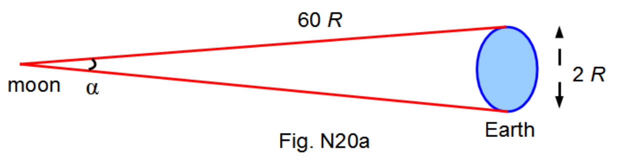

Fig. N20a

$$\alpha = \frac{2R}{60R} = \frac{1}{30} \text{ rad}$$
$$= \frac{180^{o}}{\pi}\times\frac{1}{30} = 1.9^{o} \quad .$$

(b) Diameter of moon

$$D_{moon} = (60R)\times\alpha_{moon} \quad ,$$

or $$D_{moon} = (60R)\times\left(\frac{1}{2}\right)^{o} \quad . \qquad (1)$$

Diameter of Earth

$$D_{Earth} = (60R)\times(1.9^{o}) \quad . \qquad (2)$$

Therefore, using (1) and (2)

$$\frac{D_{Earth}}{D_{moon}} = \frac{1/2^{o}}{1.9^{o}} = 3.8 \quad .$$

Thus $D_{Earth} = 3.8\,D_{moon} \approx 4\,D_{moon}$.

(c) From the given information, we find

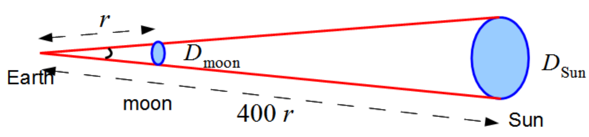

Fig. N20c

$$\frac{D_{Sun}}{D_{moon}} = \frac{400r}{r} = 400 \quad .$$

Since, from calculation of (b) part of the question, we have

$$\frac{D_{Earth}}{D_{moon}} \approx 4 \quad ,$$

we get

$$\frac{D_{Sun}}{D_{moon}}\times\frac{D_{moon}}{D_{Earth}} = 400\times\frac{1}{4} = 100 \quad ,$$

therefore,

$$\frac{D_{Sun}}{D_{Earth}} = 100 \quad .$$

N21. (a) By definition, 1 parsec is that distance at which 1 astronomical unit (A.U.) subtends an angle of 1 sec arc.

$$1 \text{ parsec} = \frac{1 \text{ A.U.}}{1''} = \frac{1 \text{ A.U.}}{4.85 \times 10^{-6}} ,$$
$$= 2.06 \times 10^{-5} \text{A.U.}$$

or $$1 \text{ parsec} \approx 2 \times 10^{5} \text{A.U.}$$

(b) Let the Sun-Earth distance is $r_{\text{S-E}}$. Then, by definition

$$r_{\text{S-E}} \approx 1 \text{ A.U.} \quad .$$

Let the diameter of the Sun is D_{Sun} . Then the angular size of the sun is

$$\alpha = \frac{D_{\text{Sun}}}{r_{\text{S-E}}} = \frac{D_{\text{Sun}}}{1 \text{ A.U.}} \quad .$$

It is given that

$$\alpha = (1/2)^{\text{o}} = \frac{D_{\text{Sun}}}{1 \text{A.U.}} \quad . \qquad (1)$$

The angular size of sun-like star at 2 parsec distance is

$$\theta = \frac{D_{\text{Sun}}}{2 \text{ pc}} = \frac{D_{\text{Sun}}}{2 \times 10^{5} \text{A.U.}} = \frac{D_{\text{Sun}}}{1 \text{ A.U.}} \times \frac{1}{2 \times 10^{5}} \quad .$$

Using the given information (Eq.(1))

$$\theta = \left(\frac{1}{2}\right)^{\text{o}} \times \left(\frac{1}{2 \times 10^{5}}\right) = 15 \times 10^{-5} \text{ min arc} \quad .$$

With magnification of 100, the angular size of star is

$$= \quad 100\, \theta = 15 \times 10^{-3} \text{ min arc} \quad .$$

This angular size is not resolvable due to atmospheric fluctuations. As a result stars are not seen magnified when viewed through a telescope.

(c) Let the diameters of the Earth and the Mars are, respectively, D_{Earth} and D_{Mars} . It is given that

$$\frac{D_{\text{Mars}}}{D_{\text{Earth}}} = \frac{1}{2} \quad , \qquad (1)$$

and the distance between the Earth and Mars,

$$r_{\text{E-M}} = \frac{1}{2} A.U. \quad . \qquad (2)$$

It is also known that (see answer of N20)

$$\frac{D_{\text{Sun}}}{D_{\text{Earth}}} = 100 \quad \text{, and} \quad \frac{D_{\text{Sun}}}{1\, A.U.} = \left(\frac{1}{2}\right)^{\text{o}} \quad . \qquad (3)$$

The angular size of the Mars is

$$\alpha_{\text{Mars}} = \frac{D_{\text{Mars}}}{r_{\text{E-M}}} = \frac{D_{\text{Mars}}}{(1/2) \text{ A.U.}} \quad . \qquad (4)$$

The above may be written as

$$\alpha_{\text{Mars}} = 2 \times \frac{D_{\text{Sun}}}{1 \text{ A.U.}} \times \frac{D_{\text{Mars}}}{D_{\text{Sun}}} \quad . \qquad (5)$$

Here,

$$\frac{D_{\text{Sun}}}{1 \text{ A.U.}} = \left(\frac{1}{2}\right)^{\text{o}} \quad \text{, and}$$

$$\frac{D_{\text{Mars}}}{D_{\text{Sun}}} = \frac{D_{\text{Mars}}}{D_{\text{Earth}}} \times \frac{D_{\text{Earth}}}{D_{\text{Sun}}} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200} \quad .$$

Using these values in Eq.(4) gives,

$$\alpha_{\text{Mars}} = 2 \times \left(\frac{1}{2}\right)^{\text{o}} \times \frac{1}{200} = \left(\frac{1}{200}\right)^{\text{o}} \quad ,$$

or $$\alpha_{\text{Mars}} = \frac{60'}{200} = 0.3' \quad .$$

With a magnification of 100, the angular size seen is

$$= \quad 100\, \alpha_{\text{Mars}} = 30' \quad .$$

This is much above the limit of resolution due to atmospheric fluctuations. Therefore, the Mars is seen magnified.

N22. (a) Oleic acid does not dissolve in water. It is dissolved in alcohol.

(b) When lycopodium powder is spread on water, it spreads on the entire surface. When a drop of the prepared solution is dropped on water, oleic acid does not dissolve in water. It spreads on the water surface pushing the lycopodium powder away to clear a circular area where the drop falls. This allow measuring the area where oleic acid spreads.

(c) $$\frac{1}{20} \frac{\text{mL} \times 1}{20} = \frac{1}{400} \text{mL} \quad .$$

(d) By means of a burette and measuring cylinder and measuring the number of drops.

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(e) If N drops of the solution make 1 mL of solution, the volume of the oleic acid in one drop will be

$$=\frac{1}{400\,N}\ \text{mL} \ .$$

N23. Density = mass / volume

$$\rho=\frac{4.237}{2.5}\ \frac{\text{g}}{\text{cm}^3}$$
$$=1.6948\ \frac{\text{g}}{\text{cm}^3} \ .$$

Since, in multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures, therefore, the density (with correct significant figures) is

$$\rho=1.7\ \text{g/cm}^3 \ .$$

N24. The length and breadth with the least count errors are, respectively,

$$L=16.2\pm0.1\,\text{cm} \ , \text{ and}$$
$$B=10.1\pm0.1\,\text{cm} \ .$$

The area is

$$A=L\,B=16.2\times10.1=163.62$$

Since, in multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures, therefore, the area up to correct significant digits is

$$A=164\,\text{cm}^2 \ .$$

The error is obtained from the following relation:

$$\frac{\Delta A}{A}=\frac{\Delta L}{L}+\frac{\Delta B}{B}=\frac{0.1}{16.2}+\frac{0.1}{10.1} \ ,$$

or
$$\frac{\Delta A}{A}=0.0062+0.0099=0.0161 \ .$$

Therefore,

$$\Delta A=164\times0.0161=2.64$$

The error in the last digit of 164 is, therefore,

$$\Delta A=3 \ .$$

Thus, the area of the sheet in appropriate significant figures and error is

$$A=164\pm3\,\text{cm}^2 \ .$$

N25. Surface Area = $2(l\,b+b\,h+l\,h)$.

Substituting values, we get

$$A=2(4.234\times1.005+1.005\times0.0201+4.234\times0.0201)$$

or
$$A=2(4.2552+0.02020+0.08510)$$
$$=2(4.3605)=8.7210\ \text{m}^2 \ .$$

Retaining result up to correct significant figures, we find

$$A=8.72\ \text{m}^2 \ .$$

The volume of the sheet is

$$V=l\,b\,h=4.234\times1.005\times0.0201$$
$$=0.085529\ \text{m}^3$$

Retaining result up to correct significant figures, we find

$$V=0.0855\ \text{m}^3 \ .$$

N26. (a) The addition of three masses, namely, 2.3 kg,
20.15 g and 20 .17 g, gives the following result:

$$\begin{array}{rl} & 2.3 \\ + & 0.02015 \\ + & \underline{0.02017} \\ & 2.34032 \end{array}$$

Since, uncertainties in subtraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted). Therefore, the total mass of the box is

2.3 kg.

(b) The difference in the masses of the pieces to correct significant figures is

20.17 – 20.15 = 0.02 g

N27. The number of significant figures in the

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measured length is 4. The calculated area and the volume should therefore be rounded off to 4 significant figures.

Surface area of the cube

$$=6(7.203)^2\,m^2$$
$$=311.299254\,m^2$$
$$=311.3\,m^2$$

Volume of the cube

$$=(7.203)^3\,m^3$$
$$=373.7147544\,m^3$$
$$=373.7\,m^3$$

N28. Density= mass/ volume

$$= \frac{5.74}{1.2}\ g/cm^3$$

$$= 4.783\ g/cm^3$$

Since, in multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures, therefore, the density (to correct significant digits) is

$$\text{Density} = 4.8\ g/cm^3$$

N29. The range of variation over the seven days of observations is

For clock 1

Δt_1 = 142 s (12:01:30 – 11:59:08 = 00:02:22 = 142 s)

For clock 2

Δt_2 = 31 s (10:15:24 – 10:14:53 = 00:00:31 = 31 s)

Although, the average reading of clock 1 is much closer to the standard time than the average reading of clock 2, but what we wish to measure is the time interval. The range of error in time interval is less for clock 2. Therefore, clock 2 should be preferred for measuring the time interval.

N30. The error (uncertain digit) in the measurements are as follows: (a) 0.01 mm, (b) 0.01 cm, (c) 0.01 m and (d) 0.01 km. Therefore, the most precise measurement is (a).

N31. The errors in the individual measurement values are:

(a) 5 – 4.9 = 0.1 cm

(b) 5 – 4.805 = 0.195 cm = 0.2 cm

(c) 5 – 5.25 = – 0.25 cm = – 0.2 cm

(d) 5 – 5.4 = – 0.4 cm

Therefore, the most precise measurement is (a).

N32. Least count of vernier calliper is

= One division of main scale – one division of vernier scale .

$$= 0.5\,mm - \frac{49\times0.5\,mm}{50},$$

$$= 0.5\,mm - 0.49\,mm,$$

$$= 0.01\,mm .$$

N33. (a) Precision tells us to what resolution or limit the quantity is measured. The precision is given by the least count of the measuring instrument. Here the least count for 20 oscillations is = 0.1 s. Therefore, the precision in the time- period (i.e., one oscillation) is

$$= \frac{0.1\,s}{20} = 0.005\,s .$$

(b) The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

The true value is taken as the mean value (in the absence of any other information).

$$t_{mean} = \frac{39.6+39.9+39.5}{3} = \frac{119}{3}\,s .$$

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Therefore, time period, $T=\frac{119}{60}\,\text{s}$

Maximum error = $\left|\frac{119}{60}-\frac{39.9}{20}\right|=0.012\,\text{s}$.

N34. The mean period of oscillation of the pendulum is

$$T=\frac{2.63+2.56+2.42+2.71+2.80}{5}\ \text{s}=\frac{13.12}{5}\ \text{s}$$

or $T=2.624\ \text{s}$
$=2.62\,\text{s}$

(Comment: As the periods are measured to a resolution of 0.01 s, all times are to the second decimal; it is proper to put this mean period also to the second decimal.)

The absolute errors in the measurements are:

$|2.62\,\text{s}-2.63\,\text{s}|=0.01\,\text{s}$

$|2.62\,\text{s}-2.56\,\text{s}|=0.06\,\text{s}$

$|2.62\,\text{s}-2.42\,\text{s}|=0.20\,\text{s}$

$|2.62\,\text{s}-2.71\,\text{s}|=0.09\,\text{s}$

$|2.62\,\text{s}-2.80\,\text{s}|=0.18\,\text{s}$

(a) The mean absolute error is

$$\Delta T=\frac{0.01+0.06+0.20+0.09+0.18}{5}\ \text{s}$$

or $\Delta T=\frac{0.54}{5}=0.11\ \text{s}$.

Comment: The mean period of oscillation may be reported as (without thinking over the mean absolute error)

$T=(2.62\pm0.11)\ \text{s}$.

However, note that the mean absolute error is in the tenth of a second, hence there is no point in reporting result up to hundredth of a second. The correct way of reporting the result of measurement is

$T=(2.6\pm0.1)\ \text{s}$.)

(b) Relative error = $\frac{\Delta T}{T_{\text{mean}}}$,

$=\frac{0.1}{2.6}=0.04$.

(c) Percentage error = Relative error × 100%

= 0.04 × 100%

= 4%

N35. Temperature difference is

$t_2-t_1=50\,^\circ\text{C}-20\,^\circ\text{C}=30\,^\circ\text{C}$.

The errors add

$\Delta t=0.5\,^\circ\text{C}+0.5\,^\circ\text{C}=1\,^\circ\text{C}$.

Therefore, the temperature difference and the error is written as

$t=30\,^\circ\text{C}\pm1\,^\circ\text{C}$.

N36. $\frac{\Delta R}{R}\%=\frac{\Delta V}{V}\%+\frac{\Delta I}{I}\%$.

Here, $\frac{\Delta V}{V}\%=\frac{5}{100}\times100=5\%$, and

$\frac{\Delta I}{I}\%=\frac{0.2}{10}\times100=2\%$.

Therefore,

$\frac{\Delta R}{R}\%=5\%+2\%=7\%$.

N37. (a) The series combination of resistances gives

$$R=(R_1\pm\Delta R_1)+(R_2\pm\Delta R_2)$$
$$=(100\pm3)\,\Omega+(200\pm4)\,\Omega$$
$$=(300\pm7)\,\Omega$$

(b) For parallel combination, the resistance is

$$R'=\frac{R_1R_2}{R_1+r_2}=\frac{100\times200}{300}=66.7\,\Omega$$.

From $\frac{1}{R'}=\frac{1}{R_1}+\frac{1}{R_2}$, we get

$$\frac{\Delta R'}{R'^2}=\frac{\Delta R_1}{R_1^2}+\frac{\Delta R_2}{R_2^2}$$,

or $$\Delta R'=\left(\frac{3}{100^2}+\frac{4}{200^2}\right)\times66.7^2$$
$$=4\times10^{-4}\times4.45\times10^3=1.78\,\Omega$$.

Therefore, for parallel combinations

$R'=(66.7\pm1.8)\,\text{ohm}$.

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Comment: (I) Since the error is in unit place of the number, the parallel combination may be better reported as

$$R'=(67\pm2)\,\text{ohm} \quad .$$

(II) A different rounding convention is sometimes followed in scientific circles when retaining significant digits in mean value and error. It requires that if three highest order digits of the error lie between 100 and 354, we round to two significant digits. Here the three highest order digits in the error are 178. So we round up to two significant digits and report the value as

$$R'=(66.7\pm1.8)\,\text{ohm} \quad .$$

N38. The relative error in Z is

$$\frac{\Delta Z}{Z}=4\frac{\Delta A}{A}+\frac{1}{3}\frac{\Delta B}{B}+\frac{\Delta C}{C}+\frac{3}{2}\frac{\Delta D}{D} \quad .$$

N39. From $T=2\pi\sqrt{L/g}$, we write

$$g=\frac{4\pi^2 L}{T^2} \quad .$$

Therefore, the relative error in g is

$$\frac{\Delta g}{g}=\frac{\Delta L}{L}+2\frac{\Delta T}{T} \quad . \qquad (1)$$

it is given that,

$$L=20.0\,\text{cm} \quad , \quad \Delta L=0.1\,\text{cm} \quad ,$$

$$T=\frac{t}{n} \text{ , therefore } \quad \Delta T=\frac{\Delta t}{n} \quad ,$$

where, t = 90 s, n = 100, Δt = 1 s. Therefore,

$$\frac{\Delta T}{T}=\frac{\Delta t}{t}=\frac{1}{90} \quad .$$

Substituting in Eq.(1), we find

$$\frac{\Delta g}{g}=\frac{0.1}{20.0}+2\times\left(\frac{1}{90}\right)=0.027 \quad .$$

The percentage error is

$$\frac{\Delta g}{g}\times100=0.027\times100=2.7$$
$$=3\% \quad .$$

N40. Observed thickness

= actual thickness × magnification

Therefore,

$$\text{actual thickness}=\frac{\text{Observed thickness}}{\text{magnification}}$$
$$=\frac{3.5\,\text{mm}}{100}=0.035\,\text{mm} \quad .$$

N41. Let the linear magnification is M_{Linear} . Then, the area magnification is

$$M_{\text{Area}}=M_{\text{Linear}}^2 \quad .$$

It is given that,

$$M_{\text{Area}}=\frac{1.55\,\text{m}^2}{1.75\,\text{cm}^2}=\frac{1.55\times10^4}{1.75}$$
$$=88.57\times10^2 \quad .$$

Therefore,

$$M_{\text{Linear}}=\sqrt{88.51}\times10=94.1 \quad .$$

N42. The relative error is

$$=\frac{\Delta t}{t}=\frac{0.02\,\text{s}}{100\,\text{yr}} \quad .$$

Since $1\,\text{yr}=3.156\times10^7\text{s}$, we get

$$\text{relative error} = \frac{0.02}{100\times3.156\times10^7} \quad .$$

or $$= 6.3\times10^{-12} \quad .$$

Thus the accuracy is 1 part in 10^{11} to 10^{12}.

N43. The value of the product is

$$A\,B=2.5(\text{m s}^{-1})\times0.10(\text{s})$$
$$=0.25\ \text{m} \quad .$$

The relative error is

$$\frac{\Delta(AB)}{(AB)}=\frac{\Delta A}{A}+\frac{\Delta B}{B}$$
$$=\frac{0.5}{2.5}+\frac{0.01}{0.10} \quad .$$
$$=0.3$$

Therefore, the product (with error) is

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$$\begin{aligned}&= AB \pm \Delta(AB)\\ &= (0.25 \pm 0.25 \times 0.3)\,\text{m}\\ &= (0.25 \pm 0.075)\,\text{m}\\ &= (0.25 \pm 0.08)\,\text{m}\end{aligned} .$$

N44. Let $C = \sqrt{AB}$. Then,

$$C = \sqrt{1.0 \times 2.0} = \sqrt{2} = 1.4$$

(up to the correct significant digits).

The relative error in C is

$$\begin{aligned}\frac{\Delta C}{C} &= \frac{1}{2}\frac{\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B}\\ &= \frac{1}{2} \times \frac{0.2}{1.0} + \frac{1}{2} \times \frac{0.2}{2.0}\end{aligned} ,$$

or $\quad \dfrac{\Delta C}{C} = 0.15$.

Therefore, the value of C is

$$\begin{aligned}C &= 1.4\,\text{m} \pm 1.4 \times 0.15\,\text{m}\\ &= 1.4\ \text{m} \pm 0.2\ \text{m}\end{aligned} .$$

Comment: We have not retained uncertainties smaller than that represented by the uncertain digit in the value 1.4 m. This is the convention of reporting error.

N45. (I) The percentage error in P is

$$\frac{\Delta P}{P}\% = 3\frac{\Delta a}{a}\% + 2\frac{\Delta b}{b}\% + \frac{1}{2}\frac{\Delta c}{c}\% + \frac{\Delta d}{d}\%$$

Substituting the values we get,

$$\frac{\Delta P}{P}\% = 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 1 \times 2\% ,$$

or $\quad \dfrac{\Delta P}{P}\% = 13\%$.

(II) The error in P is

$$\Delta P = P \times 0.13 = 3.763 \times 0.13 = 0.49 ,$$

or $\quad \Delta P = 0.5$.

Since the uncertainty is in the first decimal place, there is no point in retaining terms more uncertain than this. Therefore, the value of P that should be reported after rounding is

$= 3.8$

N46. The principle of homogeneity of dimensions states that the final dimensions on the left hand side of an equation should be equal to the final dimensions on the right hand side of an equation. Further, the argument of a trigonometric function must always be dimensionless. Therefore,

(a) In case of

$y = a \sin 2\pi t / T$,

we note that,

(i) the dimensions of the argument of the trigonometric function are

$[\,2\pi\, t/T] = [\,T^1 / T^1\,]$ = dimensionless,

and

(ii) $[y] = [L^1] = [a]$.

Therefore

DIM LHS = DIM RHS.

The relation (a) is dimensionally correct.

(b) In case of

$y = a \sin vt$,

we note that the dimensions of the argument of the trigonometric function are

$[\ vt] = [\ LT^{-1}T\ ] \equiv [L]$ =not dimensionless.

Therefore, the relation (b) is dimensionally wrong.

(c) In case of

$y = (a/T) \sin (t / a)$

we note that the dimensions of the argument of the trigonometric function are

$[\ t/a\ ] = [\ T^1 / L^1\ ]$ = not dimensionless

The relation (c) is dimensionally wrong.

(d) in case of

$y = (a\sqrt{2})\,(\sin(2\pi t / T) + \cos(2\pi t / T))$,

we note that,

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(i) The dimensions of the argument of the trigonometric function are

$$[2\pi\, t/T] = [\mathrm{T}^1 / \mathrm{T}^1] = \text{dimensionless}$$

and

(ii) $[y] = [\mathrm{L}^1] = [a]$.

Therefore,

DIM LHS = DIM RHS

The relation (d) is dimensionally correct.

N47. In the relation

$$m = \frac{m_0}{(1-v^2)^{1/2}} \quad ,$$

the dimensions of m and m_0 are the same. Therefore, from the principle of homogeneity of dimensions, the denominator on RHS should be dimensionless. Therefore, to make

$$(1-v^2)^{1/2}$$

dimensionless, we must divide the velocity v of the particle by the velocity c of light. That is, we must replace

$$(1-v^2)^{1/2} \quad \text{by} \quad (1-(v/c)^2)^{1/2} \quad .$$

Therefore, the correct relation should be

$$m = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \, .$$

N48. The dimensions of LHS are

$$[\mathrm{M}]\,[\mathrm{L\,T^{-1}}]^2 = [\mathrm{M}]\,[\mathrm{L^2\,T^{-2}}]$$
$$= [\mathrm{M\,L^2\,T^{-2}}]$$

The dimensions of RHS are

$$[\mathrm{M}][\mathrm{L\,T^{-2}}]\,[\mathrm{L}] = [\mathrm{M}][\mathrm{L^2\,T^{-2}}]$$
$$= [\mathrm{M\,L^2\,T^{-2}}]$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

N49. From the units given for the physical quantities, we note that their dimensions are as follows:

$$[K] = [\mathrm{M\,L^2\,T^{-2}}] \quad ,$$
$$[v] = [\mathrm{L\,T^{-1}}] \quad ,$$
$$[a] = [\mathrm{L\,T^{-2}}] \quad ,$$
$$[m][\mathrm{M}] \quad .$$

(a) Dimensions of LHS = $[\mathrm{M\,L^2 T^{-2}}]$

Dimensions of RHS = $[\mathrm{M}]^2[\mathrm{L\,T^{-1}}]^3 = [\mathrm{M^2 L^3 T^{-3}}]$

Since

Dim LHS ≠ Dim RHS

Therefore, formula **(a) is dimensionally ruled out.**

(b) Dimensions of LHS = $[\mathrm{M\,L^2 T^{-2}}]$

Dimensions of RHS = $[\mathrm{M}][\mathrm{L\,T^{-1}}]^2 = [\mathrm{M\,L^2 T^{-2}}]$

Since

Dim LHS = Dim RHS

Therefore, formula **(b) is dimensionally allowed.**

(Comment: The validity of the numerical factor (1/2) cannot be ascertained by dimensional analysis.)

(c) Dimensions of LHS = $[\mathrm{M\,L^2 T^{-2}}]$

Dimensions of RHS = $[\mathrm{M}][\mathrm{L\,T^{-2}}] = [\mathrm{M\,L\,T^{-2}}]$

Since

Dim LHS ≠ Dim RHS

Therefore, formula **(c) is dimensionally ruled out**.

(d) Dimensions of LHS = $[\mathrm{M\,L^2 T^{-2}}]$

Dimensions of RHS = $[\mathrm{M}][\mathrm{L\,T^{-1}}]^2 = [\mathrm{M\,L^2 T^{-2}}]$

Since

Dim LHS = Dim RHS

Therefore, formula **(d) is dimensionally allowed.**

(Comment: The validity of the numerical factor (3/16) cannot be ascertained by dimensional analysis.)

(e) Dimensions of LHS = $[\mathrm{M\,L^2 T^{-2}}]$

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Dimensions of first term of RHS

$$= [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

Dimensions of second term of RHS

$$= [M][LT^{-2}] = [MLT^{-2}]$$

Since the two terms on RHS of (e) differ in dimensions, the quantity on the right side of (e) has no proper dimensions. (Two quantities of different dimensions can not be added.)

Therefore, formula **(e) is dimensionally ruled out.**

N50. Let the dependence of time period T on the quantities l, g and m as a product may be written as :

$$T = k\, l^x g^y m^z \qquad (1)$$

where k is dimensionless constant and x, y and z are the exponents. By considering dimensions on both sides, we have

$$[L^0 M^0 T^1] = [L^1]^x [LT^{-2}]^y [M^1]^z = [L^{x+y} M^z T^{-2y}] \qquad (2)$$

On equating the dimensions on both sides, we have

$$x + y = 0; \qquad -2y = 1; \qquad z = 0$$

Therefore, on solving, we find

$$x = \frac{1}{2}\ , \quad y = -\frac{1}{2}\ , \quad z = 0\ . \qquad (3)$$

Substituting these values in Eq.(1) gives,

$$T = k\sqrt{\frac{l}{g}} \qquad (4)$$

Comment: (i) Note that value of constant k can not be obtained by the method of dimensions. Here it does not matter if some number multiplies the right side of this formula, because that does not affect its dimensions.

(ii) Actually, $k = 2\pi$ so that $T = 2\pi\sqrt{\frac{l}{g}}$.

N51. (a) Using Einstein mass energy relation,

$$E = mc^2\ ,$$

we note that the energy equivalent of 1 u is

$$E = 1.6605\times10^{-27}\times(2.9979\times10^8)\,\text{kg m}^2\text{s}^{-2} = 14.924\times10^{-11}\,\text{J}\ .$$

Using joule to MeV conversion, we find

$$E = 14.924\times10^{-11}\times\frac{1}{1.6022\times10^{-13}}\,\text{MeV} = 931.5\,\text{MeV}$$

Therefore, we write

$$1\,\text{u} = 931.5\,\text{MeV}$$

(b) One may point out that in the relation

$$1\,\text{u} = 931.5\,\text{MeV}$$

the dim of LHS are those of mass, while the dim of RHS are those of energy. So the above conversion does not exhibit correct dimensional conversion. To keep the above relationship dimensionally pure, we should write it as

$$1\,\text{u} = 931.5\,\text{MeV}/c^2\ .$$

Comment: If you use data with less number of significant figures, such as

$1\ \text{MeV} = 1.6\times10^{-13}\ \text{J}$;

$1\ \text{u} = 1.67\times10^{-27}\ \text{kg}$;

$c = 3\times10^8\ \text{m/s}$;

you will get a different answer. Try it – but do not memorize it as it represents a wrong numerical conversion factor.

N52. (a) The percentage error in the quantity X is

$$\frac{\Delta X}{X}\% = 2\frac{\Delta a}{a}\% + 3\frac{\Delta b}{b}\% + \frac{5}{2}\frac{\Delta c}{c}\% + 2\frac{\Delta d}{d}\%\ .$$

Substituting values, we get

$$\frac{\Delta X}{X}\% = 2\times1\% + 3\times2\% + \frac{5}{2}\times3\% + 2\times1\% = 12.5\%\ .$$

(b) It is given that $X = 2.763$. Therefore, the error in it is $\Delta X = 2.763\times0.125 = 0.34$.

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Since the error is in the first decimal place, the value of X should be reported as 2.8. The correct reporting (including error estimate) is

$$X = 2.8 \pm 0.3$$

N53. The dimensions of E, m, L and G are:

$$[E] = [M^1 L^2 T^{-2}] \quad ,$$
$$[m] = [M^1 L^0 T^0] \quad ,$$
$$[L] = [M^1 L^2 T^{-1}] \quad ,$$
$$[G] = [M^{-1} L^3 T^{-2}] \quad .$$

Therefore, the dimensions of P are:

$$\begin{aligned}[P] &= [M^1 L^2 T^{-2}][M^1 L^2 T^{-1}]^2 [M^1]^{-5} [M^{-1} L^3 T^{-2}]^{-2} \\ &= [M^{1+2-5+2} L^{2+4-6} T^{-2-2+4}] \\ &= [M^0 L^0 T^0]\end{aligned}$$

Thus, P is a dimensionless quantity.

N54. The dimensions of c, h and G are:

$$[c] = [L T^{-1}] \quad ,$$
$$[h] = [M L^2 T^{-1}] \quad ,$$
$$[G] = [M^{-1} L^3 T^{-2}] \quad .$$

Now consider the following combinations:

(1) $\sqrt{\dfrac{ch}{G}}$,

(2) $\sqrt{\dfrac{hG}{c^3}}$, and

(3) $\sqrt{\dfrac{hG}{c^5}}$.

Then the dimensions of these are:

(1) $$\left[\sqrt{\frac{ch}{G}}\right] = \left(\frac{[LT^{-1}][ML^2T^{-1}]}{[M^{-1}L^3T^{-2}]}\right)^{1/2} = [M^2]^{1/2} = [M] \quad .$$

It has dimensions of mass.

(2) $$\left[\sqrt{\frac{hG}{c^3}}\right] = \left(\frac{[ML^2T^{-1}][M^{-1}L^3T^{-2}]}{[LT^{-1}]^3}\right)^{1/2} = [L^2]^{1/2} = [L] \quad .$$

It has dimensions of length.

(3) $$\left[\sqrt{\frac{hG}{c^5}}\right] = \left(\frac{[ML^2T^{-1}][M^{-1}L^3T^{-2}]}{[LT^{-1}]^5}\right)^{1/2} = [T^2]^{1/2} = [T] \quad .$$

It has dimensions of time.

N55. From Kepler's third law, the square of the period of revolution T is proportional to the cube of the radius of the orbit r,

$$T^2 \propto r^3 \quad ,$$

or $$T \propto r^{3/2} \quad . \qquad (1)$$

Let us assume that T also depends on other given parameters, namely M, R and g. Let this dependence is of the form

$$T \propto g^x R^y M^z \quad . \qquad (2)$$

Combining (1) and (2), we get

$$T \propto r^{3/2} g^x R^y M^z \quad . \qquad (3)$$

Equating dimensions on the two sides, we find

$$\begin{aligned}[M^0 L^0 T^1] &= [L^{3/2}][L^x T^{-2x}][L^y][M^z] \\ &= [M^z L^{3/2+x+y} T^{-2x}]\end{aligned} \quad . \qquad (4)$$

Therefore,

$$z = 0 \quad ,$$
$$\frac{3}{2} + x + y = 0 \quad ,$$
$$-2x = 1 \quad .$$

Solving, we find

$$x = -\frac{1}{2} \quad , \quad y = -1 \quad , z = 0.$$

Therefore, using the above values in Eq.(3), we find

$$T \propto \frac{1}{R}\sqrt{\frac{r^3}{g}} \quad ,$$

or $$T = \frac{k}{R}\sqrt{\frac{r^3}{g}} \quad .$$

N56. The dimensions of LHS of the formula are

$$[\tilde{V}] = [L^3 T^{-1}] \quad . \qquad (1)$$

The dimensions of the RHS are:

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$$[\frac{\pi}{8}\frac{P r^4}{\eta l}]=\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} =[L^3T^{-1}] \quad . \quad (2)$$

Thus,

dim LHS = dim RHS

Therefore, the formula is dimensionally correct.

N57. A simple-minded guess suggests that the density should be less like that of a gas ~ 1 kg/m^3 or so. But the data reveals that

$$\text{density of the Sun} = \frac{2.0\times 10^{30}}{(4\pi/3)\times(7.0\times 10^8)^3} ,$$
$$= \frac{3\times 2.0\times 10^{30}}{4\times 3.14\times 343\times 10^{24}} ,$$
$$= 1392.7 = 1.4\times 10^3\,\text{kg/m}^3 .$$

The mass density of the Sun is in the range of densities of liquids / solids and **not** gases. This high density arises due to inward gravitational attraction on outer layers due to inner layers of the Sun.

N58. The relationship

$$\tan\theta = v$$

is dimensionally wrong. The LHS is dimensionless while the RHS has dimensions $[LT^{-1}]$. To make the relation dimensionally correct, one possible solution is to divide the RHS by some velocity, say velocity of rain fall (v_{rain}). then the dimensionally correct relation is

$$\tan\theta = \frac{v}{v_{\text{rain}}} .$$

N59. The diameter of the Na atom = 2.5Å.

Therefore, volume of the Na atom is

$$V = \frac{4\pi}{3}(1.25\times 10^{-10})^3 = 8.18\times 10^{-30}\,\text{m}^3 .$$

The mass of one atom of Na is

$$m = \frac{M}{N_A} = \frac{23}{6\times 10^{23}} = 3.8\times 10^{-23}\,\text{g} = 3.8\times 10^{-26}\,\text{kg} .$$

Therefore, the density of sodium atom is

$$\rho_{\text{Na}} = \frac{m}{V} = \frac{3.8\times 10^{-26}}{8.2\times 10^{-30}} = 4.6\times 10^3\,\text{kg/m}^3 .$$

The density of sodium in crystalline phase is (given)= $0.97\times 10^3\,\text{kg/m}^3$. The orders are same, because in the solid phase atoms are tightly packed, so the mass density of the solid is close to the atomic mass density.

(Comment: Atomic mass density is more than the mass density of solid. Although there is a close packing but still some empty space remains between the atoms of the solid. As a consequence, the solid mass density is always less than the atomic mass density.)

N60. The mass of a nucleus of mass number A is $M = A$ u (here u stands for unified atomic mass unit, $1\,\text{u} = 1.66\times 10^{-27}\,\text{kg}$). The volume of a nucleus is

$$V = \frac{4\pi r^3}{3} = \frac{4\pi r_0^3 A}{3} .$$

Therefore, density of the nucleus is

$$\rho_{\text{Nucleus}} = \frac{M}{V} = \frac{3(\text{u})}{4\pi r_0^3} .$$

It is independent of the atomic mass number, i.e., it is constant for different nuclei.

The value of nuclear density is

$$\rho_{\text{Nucleus}} = \frac{3\times 1.66\times 10^{-27}}{4\times 3.14\times 1.2^3\times 10^{-45}} ,$$

On solving we get,

$$\rho_{\text{Nucleus}} = \frac{3\times 1.66\times 10^{-27}}{4\times 3.14\times 1.2^3\times 10^{-45}} = 2.3\times 10^{17}\,\text{kg/m}^3 .$$

The nuclear density is independent of the mass number A. Therefore, this is correct estimate of the

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density of sodium nucleus.

The density of sodium atom (calculated earlier) is $4.6\times10^{3}\,\text{kg/m}^{3}$. The ratio of density of nucleus and density of atom is

$$=\frac{2.3\times10^{17}}{4.6\times10^{3}}=0.5\times10^{14}\approx10^{14}$$

Thus, nuclear density is about 10^{14} times larger than the atomic density.

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