(A) LECTURE NOTES

Based on

Physics Part-I, Textbook for Class XI, Published by NCERT

1. NEED FOR MEASUREMENT:

Measurements are needed in everyday life. Without it the simple things we do everyday would be very difficult. If we make a shirt without making measurement it is likely to be either tight or loose. If a shopkeeper gives you sugar without measuring, most likely you will not accept it. Busfare, airfare, railway-fare etc are charged on the basis of the pre-measured length of the journey to be performed. Everyone uses it in their everyday life. Measurements are needed for commerce.

Science is based on observations and deductions. Measurements are needed in science to make quantitative statements, and provide the basis for developing a theory that allow us to understand various phenomenon and make predictions.

2. FUNDAMENTAL AND DERIVED UNITS

PHYSICAL QUANTITY:

A quantity which can be measured directly or indirectly and used in explanation of physical phenomenon is called a physical quantity. For example, distance, speed, mass, pressure, force, momentum, energy.

UNIT:

A certain basic, arbitrarily chosen, internationally accepted reference standard used for measurement of a physical quantity is called a **unit**. For example, metre is a unit of length, second is a unit of time.

RESULT OF MEASUREMENT:

The result of a measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit. For example: mass of object = 5.60 kilogram, where 5.60 is the numerical measure and kilogram is the unit.

The number of physical quantities which one encounters in the study of science is large, but most of these quantities are interrelated. For example, pressure = (normal force)/area, speed = distance/time, density = mass/volume. As a result, the number of independent physical quantities which cannot be interrelated with each other is limited.

FUNDAMENTAL OR BASE UNITS:

The independent physical quantities which cannot be interrelated are called

fundamental or base quantities. To date, the number of fundamental or base physical quantities known to us is **seven** only. These are (i) length, (ii) mass, (iii) time, (iv) electric current, (v) thermodynamic temperature, (vi) luminous intensity and (vii) amount of substance.

The units for the fundamental or base quantities are called **fundamental** or **base units**.

DERIVED UNITS:

Physical quantities other than the fundamental quantities are called derived quantities. For example speed, acceleration, pressure, volume, kinetic energy are derived quantities. The units of derived quantities can be expressed as combinations of the base units. Such units are called **derived units**.

SYSTEM OF UNITS:

A **system of units** is a complete set of units which include the base units and derived units. A system of units is said to be coherent system in which all derived units are obtainable from base or fundamental units without introducing any numerical factors.

3. THE SI UNITS

SI UNITS

The **International System of Units** (abbreviated **SI** from the French *Système international d'unités*) is a system of units of measurement devised around seven base units and the convenience of the number ten.

This system of units is at present internationally accepted for measurement, both in everyday commerce and in science.

Advantage: Because SI units use decimal system, conversions within the system are simple and convenient.

The SI is not static, units are created and definitions are modified through international agreement among many nations as the technology of measurement progresses, and as the precision of measurements improves.

The SI units are divided into two classes—**base units** and **derived units**. There are seven base units, each representing, by convention, different kinds of physical quantities. These **SI base units** and their physical quantities are:

(i) metre for length

(ii) kilogram for mass

(iii) second for time

(iv) ampere for electric current

(v) kelvin for temperature

(vi) candela for luminous intensity

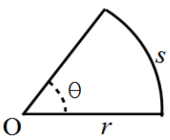
(vii) mole for the amount of substance.

There are an unlimited number of derived units formed from multiplication and division of the seven base units, for example the SI derived unit of speed is metre per second, m/s. Some derived units have special names; for example, the unit of resistance, the ohm, symbol Ω, is uniquely defined by the relation $\Omega = m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$, which follows from the definition of the quantity electrical resistance. The radian and steradian, once given special status, are now considered derived units.

Comments:

(i) PLANE AND SOLID ANGLE

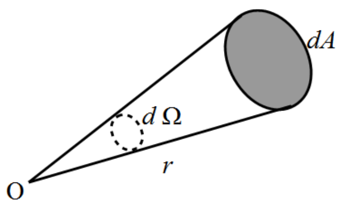
(a) The plane angle θ is defined as the ratio of length of arc *s* to the radius *r* (See Fig.1)



$$\theta = \frac{s}{r}$$

Fig.1

(b) The solid angle dΩ is defined as the ratio of the intercepted area *dA* of the spherical surface, described about the apex O as the centre, to the square of its radius *r*, as shown in Fig.2.



$$d\Omega = \frac{dA}{r^2}$$

Fig.2

The unit for plane angle is radian with the symbol rad and the unit for the solid angle is steradian with the symbol sr.

(ii) NON-SI SYSTEM OF UNITS

In earlier time scientists of different countries were using different systems of units for measurement. Three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively in the previous century. The base units for length, mass and time in these systems were as follows :

(i) In CGS system they were centimetre, gram and second respectively.

(ii) In FPS system they were foot, pound and second respectively.

(iii) In MKS system they were metre, kilogram and second respectively.)

(iii) MANAGEMENT OF SI UNITS

The **International Bureau of Weights and Measures** (French: ***Bureau international des poids et mesures***), is an international standards organisation, one of three such organisations established to maintain the International System of Units (SI) under the terms of the Metre Convention (*Convention du Mètre*). The organisation is usually referred to by its French initialism, **BIPM**. The other organisations that maintain the SI system, also known by their French initialisms are the General Conference on Weights and Measures (French: *Conférence générale des poids et mesures*) (CGPM) and the International Committee for Weights and Measures (French: *Comité international des poids et mesures*) (CIPM).

(iv) HISTORY

The metric system was conceived by a group of scientists (among them, Antoine-Laurent Lavoisier, who is known as the "father of modern chemistry") who had been commissioned by the assemblee nationale and Louis XVI of Franceto create a unified and rational system of measures. On 1 August 1793, the National Convention adopted the new decimal *metre* with a provisional length as well as the other decimal units with preliminary definitions and terms. On 7 April 1795 (*Loi du 18 germinal, an III*) the terms *gramme* and *kilogramme* replaced the former terms *gravet* (correctly *milligrave*) and *grave*. On 10 December 1799, the metric system was definitively adopted in France.

The history of the metric system has seen a number of variations, whose use has spread around the world, to replace many traditional measurement systems. At the end of World War II a number of different systems of measurement were still in use throughout the world. Some of these systems were metric-system variations, whereas others were based on customary systems. It was recognised that additional steps were needed to promote a worldwide measurement system. As a result the 9th General Conference on Weights and Measures (CGPM), in 1948, asked the International Committee for Weights and Measures (CIPM) to conduct an international study of the measurement needs of the scientific, technical, and educational communities. Based on the findings of this study, the 10th CGPM in 1954 decided that an international system should be derived from six base units to provide for the measurement of temperature and optical radiation in addition to mechanical and electromagnetic quantities. The six base units that were recommended are the metre, kilogram, second, ampere, degree Kelvin (later renamed the kelvin), and the candela. In 1960, the 11th CGPM named the system the *International System of Units*, abbreviated SI from the French name: *Le Système international d'unités*. The seventh base unit, the mole, was added in 1971 by the 14th CGPM.

4. DEFINITIONS OF SI BASE UNITS

(i) *Name:* **metre (or meter)** *Symbol:* m *Measure:* length

Definition: "The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

Definition adopted by: 17^{th} CGPM (1983).

Historical origin/justification: A metre is 1⁄10,000,000 of the distance from the Earth's equator to the North Pole measured on the circumference through Paris.

(ii) *Name:* **kilogram** *Symbol:* kg *Measure:* mass

Definition: "The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram."

*Prototype kilogram is a platinum-iridium(*90% Pt-10%Ir*) cylinder, 39 mm high and 39 mm in diameter.*

Definition adopted by: 3rd CGPM (1901).

Historical origin/justification: One kilogram is the mass of one litre of water. A litre is one thousandth of a cubic metre.


Fig.3 Prototype kilogram - image

(iii) *Name:* **second** *Symbol:* s *Measure:* time

Definition: "The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom." "This definition refers to a caesium atom at rest at a temperature of 0 K."

Definition adopted by: 13th CGPM (1967/68). The last line added by CIPM in 1997.

Historical origin/justification: The day is divided in 24 hours, each hour divided in 60 minutes, each minute divided in 60 seconds. A second is 1⁄(24 × 60 × 60) of the day.

(iv) *Name:* **ampere** *Symbol:* A *Measure:* electric current

Definition: "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length."

Definition adopted by: 9th CGPM (1948).

Historical origin/justification: The original "International Ampere" was defined electrochemically as the current required to deposit 1.118 milligrams of silver per second from a solution of silver nitrate. Compared to the SI ampere, the difference is 0.015%.

(v) *Name:* **kelvin** *Symbol:* K *Measure:* thermodynamic temperature

Definition: "The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water."

"This definition refers to water having the isotopic composition defined exactly by the following amount of substance ratios: 0.000 155 76 mole of ^{2}H per mole of

^{1}H , 0.000 379 9 mole of ^{17}O per mole of ^{16}O , and 0.002 005 2 mole of ^{18}O per mole of ^{16}O ."

Definition adopted by: 13th CGPM (1967/68). The second para added by CIPM in 2005.

Historical origin/justification: The Celsius scale: the Kelvin scale uses the degree Celsius for its unit increment, but is a thermodynamic scale (0 K is absolute zero).

(vi) *Name:* **mole** *Symbol:* mol *Measure:* amount of substance

Definition: "1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is "mol". / 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles."

"In this definition, it is understood that unbound atoms of carbon 12, at rest and in their ground state, are referred to."

Definition adopted: Adopted by 14th CGPM (1971). The last lines added by CIPM in 1980.

Historical origin/justification: Atomic weight or molecular weight divided by the molar mass constant, 1 g/mol.

(vii) *Name:* **candela** *Symbol:* cd *Measure:* luminous intensity

Definition: "The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian."

Definition adopted by:Historical origin/justification: 16th CGPM (1979).

Historical origin/justification: The candlepower, which is based on the light emitted from a burning candle of standard properties.

Summary Table of SI Base Units:

Physical quantity	Name of unit	Symbol
length	meter or metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	Candela	cd

Comments:

(i) SI PREFIXES

SI prefixes are often employed to denote decimal multiples and submultiples. The commonly used SI prefixes are :-

Name	Symbol	Value	Name	Symbol	Value
centi	c	10^{-2}	hecto	h	10^{2}
milli	m	10^{-3}	kilo	k	10^{3}
micro	µ	10^{-6}	mega	M	10^{6}
nano	n	10^{-9}	giga	G	10^{9}
pico	p	10^{-12}	tera	T	10^{12}
femto	f	10^{-15}	peta	P	10^{15}

WRITING UNIT SYMBOLS AND THE VALUES OF QUANTITIES

(i) The value of a quantity is written as a number followed by a space (representing a multiplication sign) and a unit symbol; e.g., 2.21 kg, $7.2\times10^{2}\,\mathrm{m}$, 22 K. This rule explicitly includes the percent sign (%). Exceptions are the symbols for plane angular degrees, minutes and seconds (°, ′ and ″), which are placed immediately after the number with no intervening space.

(ii) Symbols for derived units formed by multiplication are joined with a centre dot ($\cdot$) or a non-break space, for example, $N\cdot m$ or N m.

(iii) Symbols for derived units formed by division are joined with a solidus (/), or given as a negative exponent. For example, the metre per second can be written m/s , $m\,s^{-1}$, or $m\cdot s^{-1}$. Only one solidus should be used; e.g., $kg/(m\cdot s^{2})$ or $kg\,m^{-1}s^{-2}$ are acceptable but $kg/m/s^{2}$ is ambiguous and unacceptable.

(iv) Symbols are mathematical entities, not abbreviations, and do not have an appended period/full stop (.).

(v) Symbols are written in upright (Roman) type (m for metres, s for seconds), so as to differentiate from the italic type used for variables (*m* for mass, *s* for displacement).

(vi) Symbols for units are written in lower case (e.g., "m", "s", "mol"), except for symbols derived from the name of a person. For example, the unit of pressure is named after Blaise Pascal, so its symbol is written "Pa", whereas the unit itself is written "pascal".

(vii) The one exception is the litre, whose original symbol "l" is unsuitably similar to the numeral "1" or the uppercase letter "i" (depending on the typeface used), at least in many English-speaking countries. The American National Institute of Standards and Technology recommends that "L" be used instead. This has been accepted as an alternative by the CGPM since 1979. The cursive ℓ is occasionally seen, especially in

Japan and Greece, but this is not currently recommended by any standards body.

(viii) A prefix is part of the unit, and its symbol is prepended to the unit symbol without a separator (e.g., "k" in "km", "M" in "MPa", "G" in "GHz" and so on). Compound prefixes are not allowed.

(ix) All symbols of prefixes larger than 10^3 (kilo) are uppercase.

(x) Symbols of units are not pluralised; e.g., "25 kg", not "25 kgs".

(xi) Any line-break inside a number, inside a compound unit, or between number and unit should be avoided.

WRITING THE UNIT NAMES

(i) Names of units start with a lowercase letter (e.g., newton, hertz, pascal), even when the symbol for the unit begins with a capital letter. This also applies to 'degrees Celsius', since 'degree' is the unit.

(ii) Names of units are pluralised using the normal English grammar rules; e.g., "henries" is the plural of "henry". The units lux, hertz, and siemens are exceptions from this rule: they remain the same in singular and plural form. Note that this rule applies only to the full names of units, not to their symbols.)

5. SI DERIVED UNITS

The International System of Units (SI) specifies a set of seven base units from which all other units of measurement are formed. These other units are called SI derived units and are also considered part of the standard.

SI units was after the French Le Système International d'Unités which opted for a universal, unified and self-consistent system of measurement units based on the MKS (metre-kilogram-second) system.

The names of SI units are always written in lowercase. The unit symbols of units named after persons, however, are always spelled with an initial capital letter (e.g., the symbol of hertz is Hz; but metre becomes m).

DERIVED UNITS WITH SPECIAL NAMES

Base units can be combined to derive units of measurement for other quantities. In addition to the two dimensionless derived units radian (rad) and steradian (sr), 20 other derived units have special names.

Named units derived from SI base units

Name	Symbol	Quantity	Expression in terms of other units	Expression in terms of SI base units
hertz	Hz	frequency	$1/s$	s^{-1}
radian	rad	angle	$m \cdot m^{-1}$	dimensionless
steradian	sr	solid angle	$m^2 \cdot m^{-2}$	dimensionless
newton	N	force, weight	$kg \cdot m/s^2$	$kg \cdot m \cdot s^{-2}$
pascal	Pa	pressure, stress	N/m^2	$m^{-1} \cdot kg \cdot s^{-2}$
joule	J	energy, work, heat	$N \cdot m = C \cdot V = W \cdot s$	$m^2 \cdot kg \cdot s^{-2}$
watt	W	power, radiant flux	$J/s = V \cdot A$	$m^2 \cdot kg \cdot s^{-3}$
coulomb	C	electric charge or quantity of electricity	$s \cdot A$	$s \cdot A$
volt	V	voltage, electric potential difference, electromotive force	$W/A = J/C$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
farad	F	electric capacitance	C/V	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
ohm	Ω	electric resistance, impedance, reactance	V/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$
siemens	S	electric conductance	$1/\Omega$	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$
weber	Wb	magnetic flux	J/A	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
tesla	T	magnetic field strength, magnetic flux density	$V \cdot s/m^2 = Wb/m^2 = N/(A \cdot m)$	$kg \cdot s^{-2} \cdot A^{-1}$
henry	H	inductance	$V \cdot s/A = Wb/A$	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
celsius	°C	Celsius temperature	$K - 273.15$	$K - 273.15$
lumen	lm	luminous flux	$lx \cdot m^2$	$cd \cdot sr$
lux	lx	illuminance	lm/m^2	$m^{-2} \cdot cd \cdot sr$
becquerel	Bq	Radioactivity (decay per unit time)	$1/s$	s^{-1}
gray	Gy	absorbed dose (of ionizing radiation)	J/kg	$m^2 \cdot s^{-2}$
sievert	Sv	equivalent dose (of ionizing radiation)	J/kg	$m^2 \cdot s^{-2}$
katal	kat	catalytic activity	mol/s	$s^{-1} \cdot mol$

Other common units, such as the litre, are not SI units, but are accepted for use with SI.

SUPPLEMENTARY UNITS

Until 1995, the SI classified the radian and the steradian as supplementary units, but this designation was abandoned and the units were grouped as derived units.

COMPOUND UNITS DERIVED FROM SI UNITS

There exists a number compound units derived from SI units. For example:-

Name	Symbol	Quantity	Expression in terms of base SI units
square metre	m^2	area	m^2
cubic metre	m^3	volume	m^3
metre per second	m/s	speed, velocity	$m \cdot s^{-1}$
radian per second	rad/s	angular velocity	s^{-1}
…... (many more)			

UNITS OFFICIALLY ACCEPTED FOR USE WITH THE SI UNITS

Name	Symbol	Quantity	Equivalent SI unit
minute	min	time (SI unit multiple)	1 min = 60 s
hour	h	time (SI unit multiple)	1 h = 60 min = 3600 s
day	d	time (SI unit multiple)	1 d = 24 h = 1440 min = 86400 s
Degree of arc	°	Angle (dimensionless unit)	$1^{\circ} = \left(\frac{\pi}{180}\right) \text{rad}$
…..(many more)	…..	…….	…….

COMMON UNITS NOT OFFICIALLY SANCTIONED

Name	Symbol	Quantity	Equivalent SI unit
angstrom	Å	length	$1\,\text{Å} = 0.1\,\text{nm} = 10^{-10}\,\text{m}$
nautical mile	nm	length	1 nautical mile = 1852 m
bar	bar	pressure	$1\,\text{bar} = 10^5\,\text{Pa}$
….(many more)	…	….	…..

ANSWER

CONCEPTUAL QUESTIONS C1 TO C23

NUMERICAL QUESTIONS N1 TO N8

6. LENGTH MEASUREMENTS

(a) MEASUREMENT OF LARGE DISTANCES

Parallax: Parallax is an apparent displacement of an object viewed along two different lines of sight, and is measured by the angle or semi-angle of inclination between those two lines.

A simplified illustration of the parallax of an object against a distant background due to a perspective shift is illustrated in Fig.4. When viewed from "Viewpoint A", the object appears to be in front of the blank square. When the viewpoint is changed to "Viewpoint B", the object appears to have moved in front of the hatched square.

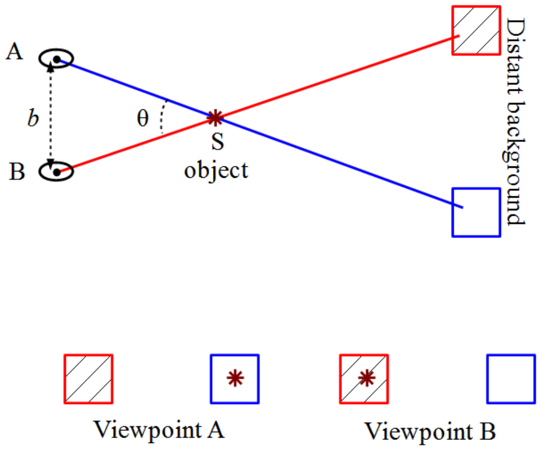


Fig.4 Understanding parallax

The "parallax" or "the parallax angle" is the angle of inclination of the two sight-lines of the distant object. In Fig.4, θ is the parallax angle. It is also known as the parallactic angle.

Basis: The distance between the two points of observation is called the basis. In Fig.4, b is the basis.

MEASUREMENT OF DISTANCE OF A FAR AWAY PLANET

To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b at the same time as shown in Fig.5.

We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB = \theta$ in Fig.5 is the parallax angle.

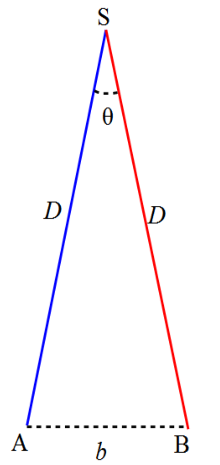


Fig.5 Parallax method

As the planet is very far away,

$$\frac{b}{D} \ll 1$$

and, therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with center at S and the distance D as the radius (D = AS = BS). Then,

$$AB = b = D\,\theta$$

where θ is in radians. Thus the distance of the distant planet is

$$D = \frac{b}{\theta} \quad .$$

MEASUREMENT OF DIAMETER OF A FAR AWAY PLANET

Let the diameter of the planet is d. Let the angular size of the planet (the angle subtended by d at the earth)is α. The distance of the planet is D. Then from the geometry of Fig.6,

$$d = \alpha D \quad . \qquad (1)$$

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through a telescope. Since D is known, the diameter d of the planet can be determined using Eq.(1).

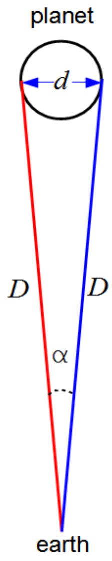


Fig.6

(b) ESTIMATION OF MOLECULAR SIZE OF OLEIC ACID

Oleic acid is a soapy liquid with large molecular size of the order of 10^{-9} m. In order to estimate the molecular size, first a mono-molecular layer of oleic acid is formed on the water surface. For this, we dissolve $1\,\text{cm}^3$ of oleic acid in alcohol to make a solution of $20\,\text{cm}^3$. Then we take $1\,\text{cm}^3$ of this solution and dilute it to $20\,\text{cm}^3$, using alcohol. So, the concentration of the solution is equal to

$$\left(\frac{1}{20\times 20}\right)\text{cm}^3$$

of oleic acid per cm^3 of solution. Next we lightly sprinkle some lycopodium powder on the surface of water in a large trough and we put one drop of this solution in the water. The oleic acid drop spreads into a thin, large and roughly circular film. Let us drop n drops in the water. These quickly spread into a circular patch. We measure the diameter of the thin film to get its area A of the patch. Suppose, volume of one drop is V (cm^3) (estimated independently). Then volume of n drops is

$$n V\ \text{cm}^3$$

The amount of oleic acid in this volume is

$$n V\left(\frac{1}{20\times 20}\right)\text{cm}^3 \quad .$$

Let the thickness of the film is t, then

$$t = \frac{\text{volume of the oleic acid circular patch}}{\text{area } A \text{ of the patch}} \quad ,$$

or
$$t = \frac{n V}{A}\left(\frac{1}{20\times 20}\right) \quad \text{cm} \quad ,$$

where V is in cm^3 and A is in cm^2. If we assume that the film has mono-molecular thickness, then this t is the size or diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9}m.

(c) RANGE OF LENGTHS

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-14} m of the tiny nucleus of an atom to the size of the order of 10^{26} m of the extent of the observable universe. Table below gives the range and order of lengths and sizes of some of these objects.

Size of object or distance	**Length (m)**	**Size of object or distance**	**Length (m)**
Size of proton	10^{-15}	Radius of the Earth	10^{7}
Size of nucleus	10^{-14}	Distance of moon from the Earth	10^{8}
Size of hydrogen atom	10^{-10}	Distance of the Sun from the Earth	10^{11}
Length of typical virus	10^{-8}	Distance of Pluto from the Sun	10^{13}
Wavelength of light	10^{-7}	Size of our galaxy	10^{21}
Size of red blood corpuscle	10^{-5}	Distance to Andromeda galaxy	10^{22}
Thickness of a paper	10^{-4}	Distance to the boundary of observable universe	10^{26}
Height of Mount Everest above sea level	10^{4}		

For measurement of such a vast range of distances, different length units are convenient. These special length units are:

1 fermi = 1 f = 10^{-15} m .

1 angstrom = 1 Å = 10^{-10} m .

1astronomical unit=1 AU (average distance of the Sun from the Earth)= 1.496×10^{11} m .

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of $3\times10^{8}\ ms^{-1}$ in 1 year).

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second).

7. MASS MEASUREMENTS

Mass is a basic property of matter. It does not depend on the temperature, pressure or location of the object in space. (However, it depends on the velocity of the object, and this dependence is observable when the speed of the object is near the speed of light.)

The SI unit of mass is kilogram (kg). However, this unit is not convenient for measuring masses of atoms and molecules. For measuring atomic and molecular masses, another standard unit of mass, called the unified atomic mass unit (u) is used.

1 unified atomic mass unit = 1u = (1/12) of the mass of an atom of carbon-12 isotope ($^{12}_{6}C$) including the mass of electrons = 1.66×10^{-27} kg

Different methods of mass measurement are employed for measuring masses in different ranges. For example, mass of commonly available objects can be determined by a common balance like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using gravitational method (that we shall study in Unit VI). For measurement of small masses of atomic/subatomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

RANGE OF MASSES

The masses of the objects, we come across in the universe, vary over a very wide range. These may vary from tiny mass of the order of 10^{-30} kg of an electron to the huge mass of about 10^{55} kg of the known universe. Table below gives the range and order of the typical masses of various objects.

Object	Mass (kg)	Object	Mass (kg)
Electron	10^{-30}	Human	10^{2}
Proton	10^{-27}	Automobile	10^{3}
Uranium atom	10^{-25}	Boeing 747 aircraft	10^{8}
Red blood cell	10^{-13}	Moon	10^{23}
Dust particle	10^{-9}	Earth	10^{25}
Rain drop	10^{-6}	Sun	10^{30}
Mosquito	10^{-5}	Milky way galaxy	10^{41}
Grape	10^{-3}	Observable universe	10^{55}

8. TIME MEASUREMENTS

Now a days, an atomic clock is used as a standard of time measurement. It is based on the measurement of periodic vibrations produced in a cesium atom. Therefore, the atomic clock is also called the cesium clock. In the cesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium-133 atom. The vibrations of the cesium atom regulate the rate of this cesium atomic clock just as the vibrations of a balance wheel regulate an ordinary wristwatch or the vibrations of a small quartz crystal regulate a quartz wristwatch.

The cesium atomic clocks are very accurate. In principle they provide portable standard. The national standard of time interval 'second' as well as the frequency is maintained through four cesium atomic clocks. A cesium atomic clock is used at the National Physical Laboratory (NPL), New Delhi to maintain the Indian standard of time. The Indian Standard Time (IST) is linked to a set of atomic clocks. The efficient cesium atomic clocks are so accurate that they impart the uncertainty in time realization as small as ± 1 part in 10^{13}. This implies that the uncertainty gained over time by such a device is less than 1 part in 10^{13}; they lose or gain no more than 3 μs in one year. In view of the tremendous accuracy in time measurement, the SI unit of length has been expressed in terms the path length light travels in certain interval of time (1/299, 792, 458 of a second).

RANGE OF TIME INTERVALS

The time interval of events that we come across in the universe vary over a very wide range, as indicated in the table below:

Event	Time interval (s)	Event	Time interval (s)
Life-span of most unstable particle	10^{-24}	Travel time for light from Sun to the Earth	10^{2}
Time required for light to cross a nuclear distance	10^{-22}	Time period of a satellite	10^{4}
Period of x-rays	10^{-19}	Rotation period of the Earth	10^{5}
Period of atomic vibrations	10^{-15}	Rotation and revolution periods of the moon	10^{6}
Period of light wave	10^{-15}	Revolution period of the Earth	10^{7}
Lifetime of an excited state of an atom	10^{-8}	Travel time for light from nearest star	10^{8}
Period of radio waves	10^{-6}	Average human life span	10^{9}
Period of sound waves	10^{-3}	Age of Egyptian pyramids	10^{11}
Wink of eye	10^{-1}	Time since dinosaurs became extinct	10^{11}
Time between successive human heart beats	10^{0}	Age of the universe	10^{17}
Travel time for light from moon to Earth	10^{0}		

Comment: Note that the ratio of the longest and shortest lengths of objects in our universe is about 10^{41} . Also, the ratio of the longest and shortest time intervals associated with the events and objects in our universe is also about 10^{41} . The ratio of the largest and smallest masses of the objects in our universe is about $\left(10^{41}\right)^2$. Is this curious coincidence between these large numbers purely accidental?

9. ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT

Error: The uncertainty in the result of a measurement (by any measuring instrument) is called error.

Error is the difference between the measured value and the true value. Every measurement is approximate due to errors in measurement. Every calculated quantity which is based on some measured values, also has an error.

Accuracy: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

Precision: Precision tells us to what resolution or limit the quantity is measured.

The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument. For example, suppose the true value of a certain length is near 3.683 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.48 cm. The first measurement has more accuracy (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

TYPES OF ERRORS

In general, the errors in measurement can be broadly classified as (a) systematic errors and (b) random errors.

SYSTEMATIC ERRORS

The systematic errors are those errors that tend to be in one direction, either positive or negative.

Some of the sources of systematic errors are :

(a) Instrumental errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc. For example, (i) the temperature graduations of a thermometer may be inadequately calibrated (it may read 101 °C at the boiling point of water at STP whereas it should read 100 °C); (ii) in a vernier calliper the zero mark of vernier scale may not coincide with the zero mark of the main scale, or (iii) simply an ordinary metre scale may be worn off at one end.

(b) Imperfection in experimental technique or procedure. For example, to determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature. Other external conditions (such as changes in temperature, humidity, wind velocity, etc.) during the experiment may systematically affect the measurement.

(c) Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc. For example, if one, by habit, always hold his/her head a bit too far to the right while reading the position of a needle on the scale, he/she will introduce an error due to parallax.

Systematic errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings.

RANDOM ERRORS

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredictable fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc), personal (unbiased) errors by the observer taking readings, etc. For example, when the same person repeats the same observation, it is very likely that he may get different readings every time.

LEAST COUNT ERROR

Least count: The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value.

The least count error is the error associated with the resolution of the instrument.

For example, a vernier callipers has the least count as 0.01 cm; a spherometer may have a least count of 0.001 cm. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. If we use a metre scale for measurement of length, it may have graduations at 1 mm division scale spacing or interval.

Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error. Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity.

10. ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

ARITHMETIC MEAN OR AVERAGE VALUE

Let the values obtained in *n* measurements are

$$a_1, a_2, \ldots, a_n \quad .$$

Then, the arithmetic mean or average value of the measurement is

$$a_{\text{mean}} = \frac{a_1 + a_2 + \ldots + a_n}{n} \quad ,$$

or

$$a_{\text{mean}} = \sum_{i=1}^{n} \frac{a_i}{n} \quad .$$

In absence of any other method of knowing true value, we take arithmetic mean as the true value. This is because it is reasonable to suppose that individual measurements are as likely to over estimate as to underestimate the true value of the quantity.

ERROR

The difference between the true value of the quantity and the individual measurement value is called the error of the measurement. In absence of any other method of knowing true value, we take arithmetic mean as the true value. The errors in the individual measurement values are:

$$\Delta a_1 = a_{\text{mean}} - a_1 \quad ,$$

$$\Delta a_2 = a_{\text{mean}} - a_2$$

… ...

… …

$$\Delta a_n = a_{\text{mean}} - a_n$$

The Δa calculated above may be positive in certain cases and negative in some other cases. The Δa is also known as the deviation.

ABSOLUTE ERROR

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement. That is the absolute value of the deviation is known as the absolute error.

In absence of any other method of knowing true value, we considered arithmetic mean as the true value. Then the absolute errors in the individual measurement values are:

$$|\Delta a_1| = |a_{\text{mean}} - a_1| \quad ,$$

$$|\Delta a_2| = |a_{\text{mean}} - a_2| \quad ,$$

… …

… …

$$|\Delta a_n| = |a_{\text{mean}} - a_n| \quad .$$

The absolute error| $|\Delta a_i|$ is always positive.

MEAN ABSOLUTE ERROR

The mean absolute error of value of a physical quantity, *a,* is defined as the arithmetic mean of all absolute errors. Thus

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_1| + \ldots + |\Delta a_n|}{n} \quad ,$$

or $$\Delta a_{\text{mean}} = \sum_{i=1}^{n} \frac{|\Delta a_i|}{n} \quad .$$

Comment: If only a single measurement of the physical quantity a is done by some one else, it is expected to be in the range

$$a_{\text{mean}} \pm \Delta a_{\text{mean}} \quad ,$$

that is, the result is expected to be in the range

$$(a_{\text{mean}} - \Delta a_{\text{mean}}) \le a \le (a_{\text{mean}} - \Delta a_{\text{mean}}) \quad .$$

RELATIVE ERROR AND PERCENTAGE ERROR

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \quad .$$

Relative error is also known as fractional error.

Percentage error is the relative error expressed in percent.

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\,\% \quad .$$

11. COMBINATION OF ERRORS

(A) ERROR OF A SUM OR A DIFFERENCE:

Rule: When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Explanation: Let two physical quantities *A* and *B* have measured value $A \pm \Delta A$ and $B \pm \Delta B$, respectively, where ΔA and ΔB are their absolute errors. Let there exist a physical quantity $Z_1 = A + B$, and another physical quantity $Z_2 = A - B$. Then the maximum value of the absolute errors in Z_1 and Z_2 are, respectively:

$$\Delta Z_1 = \Delta A + \Delta B \text{ , and}$$

$$\Delta Z_2 = \Delta A + \Delta B \text{ .}$$

The physical quantities with absolute errors are expressed as,

$$Z_1 \pm \Delta Z_1 \quad \text{and} \quad Z_2 \pm \Delta Z_2 \text{ .}$$

(B) ERROR OF A PRODUCT OR A QUOTIENT

Rule: When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

Explanation: Let two physical quantities *A* and *B* have measured value $A \pm \Delta A$ and $B \pm \Delta B$, respectively, where ΔA and ΔB are their absolute errors. Let there exist a physical quantity $Z_1 = A \times B$, and another physical quantity $Z_2 = \frac{A}{B}$. Then the maximum value of the relative errors in Z_1 and Z_2 are, respectively:

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \text{ , and}$$

$$\frac{\Delta Z_2}{Z_2} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \text{ .}$$

The physical quantities with absolute errors are expressed as,

$$Z_1 \pm \Delta Z_1 \quad \text{and} \quad Z_2 \pm \Delta Z_2 \text{ .}$$

(C) ERROR IN CASE OF A MEASURED QUANTITY RAISED TO A POWER:

Rule: The relative error in a physical quantity raised to the power *k* is the *k* times the relative error in the individual quantity.

Explanation: Let the measured values (with absolute errors) of physical quantities *A, B* and *C* are $A \pm \Delta A$, $B \pm \Delta B$ and $C \pm \Delta C$. Suppose

$$Z = \frac{A^p B^q}{C^r} \text{ .}$$

Then the relative error in Z is

$$\frac{\Delta Z}{Z} = \mathrm{p}\frac{\Delta A}{A} + \mathrm{q}\frac{\Delta B}{B} + \mathrm{r}\frac{\Delta C}{C} \quad .$$

The physical quantity Z with absolute error ΔZ is then expressed as,

$$Z \pm \Delta Z \quad .$$

12. SIGNIFICANT FIGURES

SIGNIFICANT FIGURES:

The number of significant figures indicates the precision or reliability of a measurement. The number of significant figures in a measured (reported) data is equal to the number of digits known for sure plus one that is uncertain.

A choice of change of different units does not change the number of significant digits or figures in a measurement. [For example, let the length of an object = 4.08 mm = 0.408 cm = 0.00408 m = 4080 µm. It has three significant digits.]

Rules: **(i)** All the non-zero digits are significant.

(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

(iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant. [In $\underline{0}.\underline{00}3208$, the underlined zeroes are not significant. The number of significant digits is four.]

(iv) The terminal or trailing zero(s) in a number without a decimal point are not significant. [Thus 211 m = 21100 cm = 211000 mm has ***three*** significant figures, the trailing zero(s) being not significant.] (see next rule)

(v) The trailing zero(s) in a number with a decimal point are significant. [The numbers 2.500 or 0.06800 have four significant figures each.]

Comment: (I) **Scientific notation:**

Consider the following ways of reporting measured physical quantity, say mass:

2.500 kg =2500 g

According to rule **(v)** the left hand reporting, namely 2.500 kg has four significant digits. But according to rule **(iv)** the right hand reporting, namely 2500 g has two significant digits. However, a choice of units should not change the number of significant digits. SO there appears a contradiction (fallacy!). To avoid it scientific notation is used. In this notation the reporting shall be

$$2.500\,\mathrm{kg} = 2.500 \times 10^{3}\,\mathrm{g} \quad .$$

The power of 10 is not considered in the determination of significant figures. Therefore, both, left and right hand side ways of reporting, have four significant digits.

Comment: (II) **Order of magnitude:**

Every number can be expressed as $a \times 10^b$, where *a* is a number between 1 and 10, and *b* is any positive or negative exponent (or power) of 10. In order to get an approximate idea of the number, we may round off the number *a* to 1 (for $a \le 5$) and to 10 (for $5 < a \le 10$). Then the number can be expressed approximately as 10^b in which the exponent (or power) *b* of 10 is called **order of magnitude** of the physical quantity. When only an estimate is required, the quantity is of the order of 10^b. For example, the diameter of the earth (1.28×10^7m) is of the order of 10^7m with the order of magnitude 7. The diameter of hydrogen atom (1.06×10^{-10}m) is of the order of 10^{-10} m, with the order of magnitude –10. Thus, the diameter of the earth is 17 orders of magnitude larger than the hydrogen atom.

(vi) For a number greater than 1, without any decimal, the trailing zero(s) are not significant. For a number with a decimal, the trailing zero(s) are significant.

(vii) The digit 0 conventionally put on the left of a decimal for a number less than 1 (like 0.1250) is never significant. However, the zeroes at the end of such number are significant in a measurement.

(viii) The multiplying or dividing factors which are neither rounded numbers nor numbers representing measured values are exact and have infinite number of significant digits. For example in radius $r = d/2$, or circumference $s = 2\pi r$, 2 is exact and may be taken as 2.0 or 2.000 or up to the required significant digits.

RULES FOR ARITHMETIC OPERATIONS WITH SIGNIFICANT FIGURES

(1) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

For *example*, if mass of an object is 42.27 kg and its volume 1.21 m^3, then its density (without taking care of accuracy of measurement in mass and volume) is

$$\rho = \frac{m}{V} = \frac{42.27}{1.21} = 34.9339 \text{ kg/m}^3 .$$

Here, the density should be reported as

$\rho = 34.9 \text{ kg/m}^3$ up to three (the least significant figures).

(2) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

For *example* (addition)

$$\begin{array}{r} 421.22 \\ + \quad 0.321 \\ \underline{+121.3} \\ 542.841 \end{array}$$

The above result is written ignoring the precision in various measurements. Note that the least precision is for the third term 121.3 (one place after decimal). Therefore, taking care of the significant figures, the result of addition should be written as

542.8

Furthermore, although the least number of significant figures is three in the above example (for the term 0.321) but if we write the answer as 543 (following rule 1 of multiplication/division) then its accuracy becomes less than the least precise term (namely the third term 121.3).

Similarly, for subtraction,

0.307 m – 0.304 m = 0.003 m

It has only one significant digit. It should be reported as 3×10^{-3} m . If one reports it as 3.00×10^{-3} m , it shows precision up to three figures and not up to one as reflected in the subtraction result 0.003 m. Uncertainties in subtraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

ROUNDING OFF THE UNCERTAIN DIGITS

Rules: (I) The preceding digit is raised by 1 if the insignificant digit to be dropped is more than 5, and is left unchanged if the latter is less than 5.

(II) But if the insignificant digit is 5, then the convention is that if the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1. For example, the number 7.745 rounded off to three significant figures becomes 7.74. On the other hand, the number 7.715 rounded off to three significant figures becomes 7.72 since the preceding digit is odd.

(III) In any involved or complex multi-step calculation, you should retain, in intermediate steps, one digit more than the significant digits and round off to proper significant figures at the end of the calculation.

(IV) A number known to be within many significant figures, such as in 2.99792458×10^{8} m/s for the speed of light in vacuum, is rounded off to an approximate value $3 \times 10^{8}\, m/s$, which is often employed in computations.

(V) Some constants such as $\pi = 3.1415926...$, known to have a large (infinite) number of significant figures, appear in many formulae, like

$$T = 2\pi\sqrt{\frac{L}{g}} \quad .$$

In calculations, one may take the value of π as 3.142 or 3.14 with limited number of significant figures as required in specific cases.

RULES FOR DETERMINING THE UNCERTAINTY IN THE RESULTS OF ARITHMETIC CALCULATIONS

(a) *Example:* Let the length and breadth of a plate is measured by a scale having least count of 1 mm (= 0.1 cm). The measured values are *L*= 16.2 cm, and *B* = 10.1 cm. This means that (including possible uncertainty due to least count)

$L = 16.2 \pm 0.1$ cm , and $B = 10.1 \pm 0.1$ cm ,

or $L = 16.2$ cm $\pm 0.6\%$, and $B = 10.1$ cm $\pm 1\%$.

The area of the plate is, then, (ignoring the rules for significant digits)

$L \times B = 163.62$ cm$^2 \pm 1.6\%$,

or $L \times B = 163.62 \pm 2.6$ cm^2 .

Now applying the rules for significant figures (noticing that *L* and *B* are given up to three significant digits), we write

$L \times B = 164 \pm 3$ cm^2 .

Here 3 cm^2 is the uncertainty or error in the estimation of area of rectangular sheet.

(b) If a set of experimental data is specified to *n* significant figures, a result obtained by combining the data will also be valid to *n* significant figures.

Uncertainties in subtraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

Example: Consider the subtraction given below-

$$\begin{array}{r} 13.9 \\ \underline{-7.05} \\ 6.84 \end{array}$$

This is not reported as 6.84 because it will mean precision more than the least (for 13.9). Therefore, the result of the subtraction should be reported as 6.8 .

(c) The relative error of a value of number specified to significant figures depends not only on *n* but also on the number itself.

Example: Consider measurements of two masses:

$m_1 = 1.02 \pm 0.01$ g , and

$m_2 = 9.89 \pm 0.01$ g .

Although the precision is the same in the two measurements, but the relative errors are quite different:

$$\frac{\Delta m_1}{m_1}\times 100=\frac{0.01}{1.02}\times 100=1\%$$, and

$$\frac{\Delta m_2}{m_2}\times 100=\frac{0.01}{9.89}\times 100=0.1\%$$.

(d) Intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement, otherwise rounding errors can build up.

Example: Consider reciprocal of 9.49 . Retaining three significant figures gives

$$\frac{1}{9.49}=0.105$$.

Then, we find that the reciprocal of 0.105 deviates much from the value 9.49. In fact

$$\frac{1}{0.105}=9.52$$ (up to three significant figures).

However, if one retains one more significant figures in the intermediate calculation, the rounding errors reduce to a great extent:

$$\frac{1}{9.49}=0.1054 \quad \text{and} \quad \frac{1}{0.1054}=9.49$$.

ANSWER

CONCEPTUAL QUESTIONS C24 TO C58

NUMERICAL QUESTIONS N9 TO N49

13. DIMENSIONS OF PHYSICAL QUANTITIES

At present, there are seven fundamental or base physical quantities required for the description of all derived physical quantities of the physical world. These seven base quantities are known as the seven dimensions of the physical world. We use square bracket [ ] to denote the dimensions. Thus, length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd], and amount of substance [mol]. All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental or base dimensions.

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

Examples:

(i) volume of an object $V=l\times b\times h=(\text{length})^3$, thus $[V]=[L^3]$. Volume is independent of mass and time, so we may write

$[V]=[M^0L^3T^0]$.

The volume possess zero dimension in mass, zero dimension in time and three dimensions in length.

(ii) velocity (v) = displacement/time. Therefore, $[v]=[M^0L^1T^{-1}]$. The velocity has zero dimension in mass, one dimension in length and -1 dimension in time.

14. DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

DIMENSIONAL FORMULA

The dimensional formula of a given physical quantity is the expression which shows how and which of the base quantities represent the dimensions of that physical quantity.

Examples: The dimensional formula (i) of volume is $[M^0L^3T^0]$, (ii) of speed or velocity is $[M^0L^1T^{-1}]$, (iii) of acceleration is $[M^0L^1T^{-2}]$, (iv) of mass density is $[M^1L^{-3}T^0]$.

DIMENSIONAL EQUATION

An equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation** of the physical quantity.

The dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities.

Examples: The dimensional equations of volume [V], speed [v], force [F] and mass density [ρ] may be expressed as

$[V]=[M^0L^3T^0]$,

$[v]=[M^0L^1T^{-1}]$,

$[F]=[M^1L^1T^{-2}]$,

$[\rho]=[M^1L^{-3}T^0]$.

(Many a times the exponent 1 is not written explicitly, i.e, one may write $[F]=[MLT^{-2}]$.)

The dimensional equation can be obtained from the equation representing the relations between the physical quantities.

The dimensional formulas of some of physical quantities are given in the Table below. (Dimensions and dimensional equations is normally mentioned in these notes in various chapters where a physical quantity is encountered for the first time.) (You may browse Appendix 9 of the NCERT Book also to get a feel of different physical quantities you will encounter during your XI and XII.)

S.No.	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional Formula
1	Area	Length × breadth	$[L^2]$	$[M^0L^2T^0]$
2	Volume	Length × breadth × height	$[L^3]$	$[M^0L^3T^0]$
3	Mass density	Mass / Volume	$[M/L^3]$	$[ML^{-3}T^0]$
4	Frequency	1/Time	$1/[T]$	$[M^0L^0T^{-1}]$
5	Velocity	Displacement / Time	$[L]/[T]$	$[M^0LT^{-1}]$
6	Acceleration	Velocity / Time	$[LT^{-1}]/[T]$	$[M^0LT^{-2}]$
7	Force	Mass × Acceleration	$[M][LT^{-2}]$	$[MLT^{-2}]$
8	Impulse	Force × Time	$[MLT^{-2}][T]$	
9	Work, Energy	Force × Distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
10	Power	Work / Time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
11	Momentum	Mass × Velocity	$[M][LT^{-1}]$	$[MLT^{-1}]$
12	Pressure, stress	Force / Area	$[MLT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
13	Strain	$\frac{\text{change in dimension}}{\text{original dimension}}$	$[L]/[L]$ or $[L^3]/[L^3]$	$[M^0L^0T^0]$
14	Modulus of elasticity	Stress / Strain	$[ML^{-1}T^{-2}]/[M^0L^0T^0]$	$[ML^{-1}T^{-2}]$
15	Surface tension	Force / Length	$[MLT^{-2}]/L$	$[ML^0T^{-2}]$
16	Angular displacement	Arc/ Length	$[L]/[L]$	$[M^0L^0T^0]$
17	Angular velocity	Angular displacement/Time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$
18	Torque	Force × distance	$[MLT^{-2}] \times [L]$	$[ML^2T^{-2}]$
19	Angular acceleration	Angular velocity / Time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
20	Moment of Inertia	Mass×(radius of gyration)^2	$[M] \times [L^2]$	$[ML^2T^0]$
21	Angular momentum	Moment of Inertia × (angular velocity)	$[ML^2] \times [T^{-1}]$	$[ML^2T^{-1}]$

15. DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

(A) CHECKING THE DIMENSIONAL CONSISTENCY OF EQUATIONS

The principle of homogeneity of dimensions states that the final dimensions on the left hand side of an equation should be equal to the final dimensions on the right hand side of that equation.

Explanation: The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. If the dimensions of all the terms are not same, the equation is wrong. Dimensions are customarily used as a preliminary test of the consistency of an equation, when there is some doubt about the correctness of the equation.

It may be noted that a test of consistency of dimensions tells us no more and no less than a test of consistency of units, but has the advantage that we need not commit ourselves to a particular choice of units, and we need not worry about conversions among multiples and sub-multiples of the units.

It must be remembered that if an equation fails dimensional consistency test, it is proved wrong, but if it passes, it is not proved right. Thus, a dimensionally correct equation need not be actually an exact (correct) equation, but a dimensionally wrong (incorrect) or inconsistent equation must be wrong.

Illustration: Question: The time period T of a simple pendulum of length L is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \quad , \qquad ...(1)$$

where $g = 9.80\,\mathrm{m/s^2}$ is acceleration due to gravity. Check dimensionally its consistency.

Answer: The dimensions of the LHS are

$$[T] = [\mathrm{M^0\,L^0\,T^1}] \qquad ...(2)$$

The dimensions of right hand side are

$$[2\pi\sqrt{L/g}] = [\mathrm{L^{1/2}/(L\,T^{-2})^{1/2}}] = [\mathrm{T}] \equiv [\mathrm{M^0\,L^0\,T^1}] \qquad ...(3)$$

Comparing (2) and (3) we note that Eq.(1) is dimensionally consostent.

(B)DEDUCING RELATION AMONG THE PHYSICAL QUANTITIES

The method of dimensions can sometimes be used to deduce relation among the physical quantities. For this we should know the dependence of the physical quantity on other quantities (up to three physical quantities or linearly independent variables) and consider it as a product type of the dependence.

*Illustration: Question:*The speed of sound in a medium is expected to depend on the elasticity of the medium and the inertia of the medium. Using dimensional analysis, find the possible dependence.

Answer: Let the speed is V, elastic constant of the medium is E and the mass density of the medium is ρ . The dependence is assumed to be of the form

$$V \propto E^a \rho^b \quad ,$$

or $$V = k\,E^a \rho^b \quad . \qquad ...(1)$$

where k is assumed to be a dimensionless constant.

The dimensions of the physical quantities involved are:

$$[V] = [\mathrm{L\,T^{-1}}] \quad , \quad [E] = [\mathrm{M\,L^{-1}\,T^{-2}}] \quad \text{, and} \quad [\rho] = [\mathrm{M\,L^{3}}] \quad . \qquad ...(2)$$

By considering dimensions on both the sides of Eq.(1), we have

$$[\mathrm{L\,T^{-1}}] = [\mathrm{M\,L^{-1}\,T^{-2}}]^a [\mathrm{M\,L^3}]^b \quad ,$$

or $$[\mathrm{M^0\,L\,T^{-1}}] = [\mathrm{M^a\,L^{-a}\,T^{-2a}}][\mathrm{M^b\,L^{3b}}] \quad ,$$

or $$[\mathrm{M^0\,L\,T^{-1}}] = [\mathrm{M^{a+b}\,L^{-a+3b}\,T^{-2a}}] \quad .$$

Equating the dimensions on both the sides, we get

$$0=a+b \quad , \quad ...(3)$$

$$1=-a+3b \quad ,...(4)$$

and $$-1=-2a \quad . \quad ...(5)$$

Solving the above set of equations, we find

$$a=\frac{1}{2} \quad \text{and} \quad b=-\frac{1}{2} \quad .$$

Using the above in eq.(1) gives

$$V=k\sqrt{\frac{E}{\rho}} \quad .$$

(The constant cannot be determined by the dimensional analysis).

(C) CONVERTING THE NUMERICAL MEASURE OF A PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER SYSTEM OF UNITS

The result of a measurement of a physical quantity is expressed by a number n (or numerical measure) accompanied by a unit u. If two different systems of units are used, their corresponding numerical measures will be different, but the following relation will hold:

$$n_1 u_1 = n_2 u_2 \quad . \quad ...(1)$$

Let the dimensions of a physical quantity Q are

$$[Q]=[\mathrm{M}^a \mathrm{L}^b \mathrm{T}^c] \quad . \quad ...(2)$$

Then, using Eq.(1) we can write,

$$n_2 = n_1\left[\left(\frac{\mathrm{M}_1}{\mathrm{M}_2}\right)^a\left(\frac{\mathrm{L}_1}{\mathrm{L}_2}\right)^b\left(\frac{\mathrm{T}_1}{\mathrm{T}_2}\right)^c\right] \quad . \quad ...(3)$$

This equation converts the numerical measure n_1 to n_2 .

Illustration: Question: The unit of force in SI units is newton, and the unit of force in cgs system of units is dyne. Find the numerical measure required for conversion of newton to dyne.

Answer: The dimensions of force are

$$[F]=[\mathrm{M}^1 \mathrm{L}^1 \mathrm{T}^{-2}] \quad .$$

If n_1 and n_2 numerical measures, respectively, in SI units and cgs units, then

$$n_2 = n_1\left[\left(\frac{\mathrm{M}_1}{\mathrm{M}_2}\right)^1\left(\frac{\mathrm{L}_1}{\mathrm{L}_2}\right)^1\left(\frac{\mathrm{T}_1}{\mathrm{T}_2}\right)^{-2}\right] \quad ,$$

or $$n_2 = 1\times\left[\left(\frac{\mathrm{kg}}{\mathrm{g}}\right)^1\left(\frac{\mathrm{m}}{\mathrm{cm}}\right)^1\left(\frac{\mathrm{s}}{\mathrm{s}}\right)^{-2}\right] \quad ,$$

or $n_2 = 1 \times \left[(1000)^1 (100)^1 (1)^{-2}\right]$,

or $n_2 = 10^5$.

Therefore, using the above information in $n_1 u_1 = n_2 u_2$, we get

$1\,\text{newton} = 10^5\,\text{dyne}$.

(D) LIMITATIONS OF DIMENSIONAL ANALYSIS

(i) The method of dimensional analysis gives no information about any dimensional constant and any numerical factor appearing in equation.

(ii) If a physical quantity depends on more than three variables, then methods of dimensional analysis can not be used.

(iii) If a physical quantity involves trigonometric functions or logarithmic functions, such as $y = A\sin(\omega t)$, $T = (\log_e 2)/\lambda$, then the dimensional analysis can not be used for deducing relation among physical quantities.

(iv) The methods of dimensional analysis can not be used to establish relations involving addition or subtraction of two or more terms (e.g., $s = ut + \frac{1}{2}at^2$).

(v) If a relation contains an unknown dimensional constant, then the methods of dimensional analysis fails.

ANSWER

CONCEPTUAL QUESTIONS C59 TO C75

NUMERICAL QUESTIONS N46 TO N60