At fundamental constituent level, a particle which does not exhibit weak interactions, does not have rest mass.

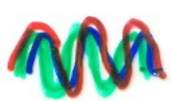
Sardar Singh

- 1. Neutrinos produced/annihilated in weak interactions do not have definite rest mass.
- 2. At fundamental constituent level, a particle which does not exhibit weak interactions, does not have rest mass.
- 3. At fundamental constituent level, a particle may have one / two / three different kinds of charges, with differing algebras, leading to three different fundamental interactions (i.e., a particle having two kinds of charges exhibit two kinds of interactions).

- 4. When strength of the interaction depends on helicity of any one of the interacting particles, the PARITY and CHARGE CONJUGATION symmetries are violated.
- 5. Only that Lagrangian density of fields can give observable interactions for which the ACTION is invariant under some local gauge transformation of fields.
- 6. When various charges are expressed in a dimensionless manner, their relative magnitudes are of the same order.

A Neutrino does not have a definite rest mass ? ☹

Consider $\pi^+ \rightarrow \mu^+ \nu_\mu$. I am not claiming $\pi^+ \rightarrow \mu^+ \nu_1$ or $\pi^+ \rightarrow \mu^+ \nu_2$ etc. But I mean



$$|\nu_\mu(0)\rangle = \sum_j C_{\mu j} \, |\nu_j(0)\rangle$$

As time passes and the neutrino (packet) moves, each mass eigenstate picks up a phase….

$$e^{-i(E_j t - p_j x)}$$

$$\approx e^{-i(m_j^2 L/2E)}$$

$$E = \sqrt{p^2 + m^2}$$

$$\simeq p + \frac{m^2}{2p}$$

$$E \sim p$$

$$x \sim ct$$

As a consequence, at a distance L,

$$|\nu_\mu(L)\rangle = \sum C_{\mu j}\, e^{-i\, m_j^2 L/2E}\, |\nu_j(0)\rangle$$

Find probability amplitude

$$\langle \nu_e(0) | \nu_\mu(L) \rangle = \sum_j C^*_{ej} \, C_{\mu j} \, e^{-i m_j^2 L/2E}$$

The Probability

$P(\nu_\mu \longrightarrow \nu_e ; L)$ disappearance

$P(\nu_\mu \longrightarrow \nu_\mu ; L)$ retention of original flavour.

In a “toy” model

$$\begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \end{pmatrix} = \begin{pmatrix} \cos\theta_{e\mu} & \sin\theta_{e\mu} \\ -\sin\theta_{e\mu} & \cos\theta_{e\mu} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}$$

$$P(\nu_e \to \nu_\mu ; L) = \sin^2 2\theta_{e\mu} \, \sin^2(\Delta m^2_{21} L/4E)$$

with $\Delta m^2_{21} \equiv m_2^2 - m_1^2$,

$$P(\nu_e \to \nu_e ; L) = 1 - P(\nu_e \to \nu_\mu ; L)$$

Require volunteer to work on

“BEYOND TOY MODEL”

A particle which does not exhibit weak interaction does not have rest mass ? ☹

In Standard Model of particle physics:

12 flavours of particles build matter

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

12 gauge fields mediate interactions

$$\gamma, W^\pm, Z^0, g_1, \cdots g_8$$

Each quark flavour is of three “colour” kinds “R, B, G”….

Degenerate for EM and weak interactions…

Distinct only for the strong interaction…

The Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ Lagrangian density is

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a} \\
& + \bar{L}\gamma^{\mu}\left(i\partial_{\mu} - \frac{1}{2}g\,\vec{\tau}\cdot\vec{W}_{\mu} - \frac{1}{2}g' Y B_{\mu}\right)L \\
& + \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - \frac{1}{2}g' Y B_{\mu}\right)R \\
& + \left|\left(i\partial_{\mu} - \frac{1}{2}g\,\vec{\tau}\cdot\vec{W}_{\mu} - \frac{1}{2}g' Y B_{\mu}\right)\phi\right|^{2} - V(\phi) \\
& - \left(g_{1}\bar{L}\phi R + g_{2}\bar{L}\tilde{\phi}R + \text{hermitian conjugate}\right) \\
& + \frac{1}{2}g_{s}\left(\bar{\psi}^{j}_{q}\gamma^{\mu}\lambda^{a}_{jk}\psi^{k}_{q}\right)G^{a}_{\mu}
\end{aligned}
$$

symbols:

L left-handed fermion doublet,

R right-handed fermion singlet,

ϕ ($\tilde{\phi}$) Higgs doublet and its conjugate

ψ_q^j quark colour fields

A mass term in the Lagrangian density, which is of the form

$$\bar{\psi} M \psi = \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$

can not be accommodated in the SM…

Reason: EM interaction …same for ψ_L, ψ_R

Colour interaction same for ….. ψ_L, ψ_R

But weak interaction …not the same for ψ_L, ψ_R

Therefore a mass term $\bar{\psi} M \psi$ spoils $SU(2)_L$
Symmetry (invariance of the action).

Conclusion: If a particle can undergo weak interaction, then a mass term can not be accommodated in the action.

All the 12 flavours constituting matter undergo weak interactions.

The three gauge bosons, W^{+}, W^{-} ,and Z^{0} are also massive.

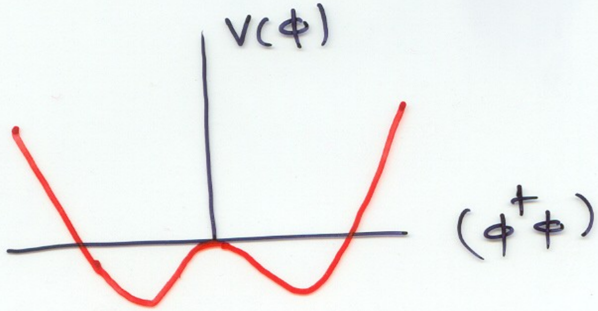
The remaining gauge bosons, one photon and the eight gluons do not participate in the weak interactions, and are mass less.

Thus the mechanism for giving mass to a particle has to be that suggested in the weak interaction models….the HIGGS MECHANISM.

HIGGS MECHANISM ?

Fundamental scalar "Higgs" fields interact and acquire a nonzero vacuum expectation value (breaking the $SU(2)_L X\ U(1)_Y$ symmetry, separating weak and EM interactions). The W^+,W^-, Z^0 and fermions acquire mass by interacting with the vacuum Higgs fields

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad (\text{with } \mu^2 < 0 \text{ and } \lambda > 0)$$



"Mexican hat"

minimum at $\phi^{\dagger}\phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$

These considerations lead to

$$m_{fermion} \propto \frac{v}{\sqrt{2}}$$

$$= g_f \frac{v}{\sqrt{2}}$$

The minimum version needs one Higgs doublet fields leading to one Higgs boson of zero charge but finite mass. The SM predicts

$$M_W = \frac{1}{2} g v,$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$m_H = \sqrt{2\lambda v^2} = ?$$

$$v = \left(\frac{1}{\sqrt{2}G}\right)^{1/2} \simeq 246 \text{ GeV}$$

$$m_{fermion} = g_f \frac{v}{\sqrt{2}}$$

??!!

THANKS