Problem 9

In the CM frame, for the process A + B → C + D, show that

(i) $F = 4\, p_i \sqrt{s}$, where $|p_A| = |p_B| = p_i$, and $s=(E_A+E_B)^2$.

(ii) $dQ = \frac{1}{4\pi^2}\frac{p_f}{4\sqrt{s}}\,d\Omega$ **, where $|p_C| = |p_D| = p_f$ and $d\Omega$ is the element of solid angle about p_C .**

$$\therefore \left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|M|^2 .$$

(i) In CM frame $\mathbf{p}_A = -\mathbf{p}_B$, and for any collinear collision,

$$F = 4\sqrt{(p_A . p_B)^2 - m_A^2 m_B^2}$$

$$= 4\,(|\mathbf{p}_A|\, E_B + |\mathbf{p}_B|\, E_A)$$

$$= 4\, p_i\,(E_A + E_B) = 4\, p_i \sqrt{s} .$$

(ii) $$dQ = (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \times \frac{d^3 p_C}{(2\pi)^3 2E_C} \times \frac{d^3 p_D}{(2\pi)^3 2E_D}$$

$$= \frac{1}{(2\pi)^2}\frac{\delta(E_C + E_D - E_A - E_B)}{2E_C 2E_D} \times d^3\vec{p}_C \delta(\vec{p}_C + \vec{p}_D - \vec{p}_A - \vec{p}_B) d^3\vec{p}_D$$

Integrate over $d^3\vec{p}_D$ to get

$$dQ = \frac{1}{4\pi^2}\frac{\delta(E_C + E_D - E_A - E_B)}{2E_C 2E_D} d^3\vec{p}_C$$

with $\vec{p}_D = \vec{p}_A + \vec{p}_B - \vec{p}_C$ and $E_D = \sqrt{|\vec{p}_D|^2 + m_D^2}$.

Now $d^3\vec{p}_C = |\vec{p}_f|^2 d|\vec{p}_f| d\Omega$.

Let $W = E_C + E_D = \sqrt{m_C^2 + |\vec{p}_f|^2} + \sqrt{m_D^2 + |\vec{p}_f|^2}$.

This implies

$$\frac{dW}{d|\vec{p}_f|} = |\vec{p}_f| \frac{(E_C + E_D)}{E_C E_D}$$

or $\quad d|\vec{p}_f| = \frac{dW \; E_C E_D}{|\vec{p}_f| W}.$

$$\therefore \; dQ = \frac{1}{4\pi^2} \times \frac{\delta(W - \sqrt{s})}{4} \times \frac{|\vec{p}_f|}{W} \times dW \times d\Omega \,.$$

Integrate over dW to get

$$dQ = \frac{1}{4\pi^2} \times \frac{|\vec{p}_f|}{4\sqrt{s}} \times d\Omega \,.$$

Substitute the values of F and dQ to get $d\sigma$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 s} \quad \frac{p_f}{p_i} \quad |M|^2 \quad .$$