Problem 6

Consider $e^- \mu^-$ scattering: $e^-(A) + \mu^-(B) \rightarrow e^-(C) + \mu^-(D)$. Write the transition amplitude. Draw Feynman diagram in configuration space. Identify the invariant amplitude and draw Feynman diagram in momentum space.

The transition amplitude for $e^- \mu^-$ scattering is

$$T_{fi} = -i\int d^4x\, j_\mu^{(1)}(x)\, A^\mu_{(2)}(x)$$

$$= -i\int d^4x\, j_\mu^{(1)}(x)\, (-\frac{1}{q^2})\, j^\mu_{(2)}(x)$$

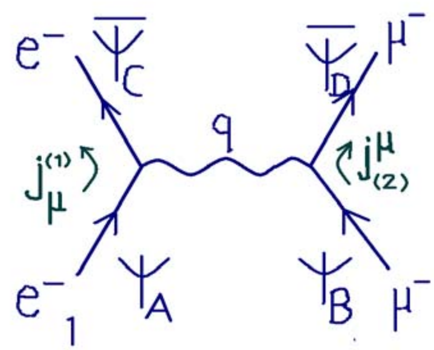
where (1) refers to the electron and (2) refers to the muon. The q^2 is the suare of four-momentum transfer. The currents are

$$j_\mu^{(1)}(x) = -e\,\overline{\psi}_C(x)\,\gamma_\mu\,\psi_A(x),$$

$$j^\mu_{(2)}(x) = -e\,\overline{\psi}_D(x)\,\gamma^\mu\,\psi_B(x).$$

The diagrammatic representation of

The transition amplitude is as shown (see→).



Using $\psi(x) = u(\mathbf{p})\, e^{-i\,p\,.\,x}$, one gets

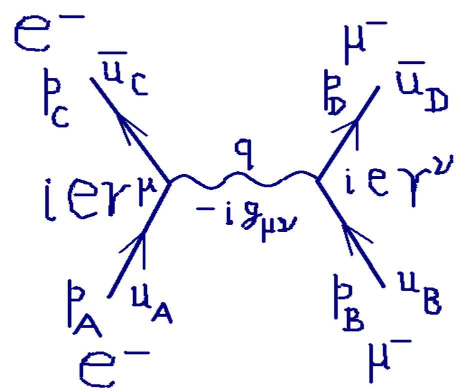
$$T_{fi} = -i\left[-e\bar{u}_C\gamma_\mu u_A\right]\left[-\frac{1}{q^2}\right]\left[-e\bar{u}_D\gamma^\mu u_B\right]\times\int d^4x\ \ \exp\{i(p_D + p_C - p_A - p_B)\}$$

or $T_{fi} = -i\,M\,(2\pi)^4\,\delta^4(p_D + p_C - p_A - p_B)$

where M is called the invariant amplitude. Here

$$-iM = \left[\bar{u}_C(ie\gamma^{\mu})u_A\right]\left[\frac{-ig_{\mu\nu}}{q^2}\right]\left[\bar{u}_D(ie\gamma^{\nu})u_B\right].$$

Diagrammatically it is represented as below. The diagram is called Feynman diagram in momentum space.



$$q = p_A - p_C = p_D - p_B$$

Comment: Normalization constants are included in the spinors.