Problem 5

Imagine EM interaction in electron-muon scattering. Let the electron is interacting with the EM field $A^\mu(x)$ whose source is the muon. What is the nature of the $A^\mu(x)$?

Let the source of $A^\mu(x)$ is the muon (particle 2). That is

$$\Box^2 A^\mu(x) = j^\mu_{(2)}(x),$$

where $j^\mu_{(2)}(x) = -e\,\bar{u}_f(\vec{p}_f)\gamma^\mu u_i(\vec{p}_i)\,e^{i\,q.x}$ with $q = p_f - p_i$, four-momentum transfer.

Note that we may write (use $\Box^2 \; (\Box^2)^{-1} = 1$)

$$A^\mu(x) = (\Box^2)^{-1}\; j^\mu_{(2)}(x).$$

Now

$$\Box^2\, e^{i\,q\,.\,x} = \partial^\mu\partial_\mu e^{iq.x} = -q^\mu q_\mu e^{iq.x} = -q^2 e^{iq.x}$$

$$\therefore\; (\Box^2)^{-1}\, e^{i\,q\,.\,x} = -\,(1/q^2)\, e^{i\,q\,.\,x}$$

or $\quad A^\mu(x) = (\Box^2)^{-1}\; j^\mu_{(2)}(x)$

$$= -\frac{1}{q^2}\; j^\mu_{(2)}(x).$$

Comment:

If you ignore the spin of the particles and consider “spin-less” electron-muon scattering, then in this case too,

$$A^\mu(x) = -\frac{1}{q^2}\; j^\mu_{(2)}(x)$$

Here the spin-less muon current is of the form

$j^\mu_{(2)}(x) = -\,e\,N_i\,N_f\,(p_i + p_f)^\mu\;\exp\{i\,(p_f - p_i)\,.\,x\}$, with $q = p_f - p_i$. Hint: See § 4.2 : Quarks and Leptons, Halzen and Martin, John Wiley & Sons © 1984