Problem 14

Show that (it is called Gordon decomposition)

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f [(p_i + p_f)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu] u_i \, .$$

- We write

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m}[(m\bar{u}_f)\gamma^\mu u_i + \bar{u}_f \gamma^\mu (m u_i)] .$$

The Dirac equation gives

$$\gamma^\mu p_\mu u(\vec{p}) = m u(\vec{p}) \, ,$$

and $\bar{u}(\vec{p})\gamma^\mu p_\mu = m\bar{u}(\vec{p})$.

Therefore, we may write

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m}[\bar{u}_f \gamma^\nu p_{f\nu} \gamma^\mu u_i + \bar{u}_f \gamma^\mu \gamma^\nu p_{i\nu} u_i] .$$

Use $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$, to get

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m}[\bar{u}_f p_{f\nu}(2g^{\mu\nu} - \gamma^\mu \gamma^\nu) u_i + \bar{u}_f \gamma^\mu \gamma^\nu p_{i\nu} u_i]$$

$$= \frac{1}{2m}[2\bar{u}_f p_f^\mu u_i - \bar{u}_f \gamma^\mu \gamma^\nu (p_f - p_i)_\nu u_i] .$$

Similarly one may also write:

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m}[\bar{u}_f p_{f\nu} \gamma^\nu \gamma^\mu u_i + \bar{u}_f (2g^{\mu\nu} - \gamma^\nu \gamma^\mu) p_{i\nu} u_i]$$

$$= \frac{1}{2m}[2\bar{u}_f p_i^\mu u_i - \bar{u}_f (-\gamma^\nu \gamma^\mu)(p_f - p_i)_\nu u_i] .$$

Adding the “two”, we get

$$2\bar{u}_f \gamma^\mu u_i = \frac{1}{2m}[2\bar{u}_f (p_f + p_i)^\mu u_i - \bar{u}_f (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)(p_f - p_i)_\nu u_i],$$

or

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f [(p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu] u_i \quad \prec$$