Problem 13

Show that for very high-energy "spin-less" e – μ scattering

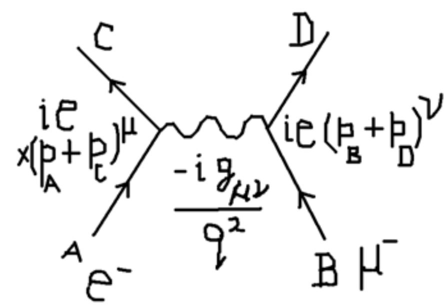
$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

where θ = scattering angle, and $\alpha = e^2 / 4\pi$. Very high-energy here means the particle masses are neglected.

➢ The Feynman diagram in momentum space, for the spin-less e – μ scattering is shown here. The invariant amplitude M is

$$-iM = ie(p_A + p_C)^{\mu}\frac{-ig_{\mu\nu}}{q^2}ie(p_B + p_D)^{\nu},$$

$$\therefore\ |M|^2 = \frac{e^4}{q^4}\{(p_A + p_C).(p_B + p_D)\}^2.$$



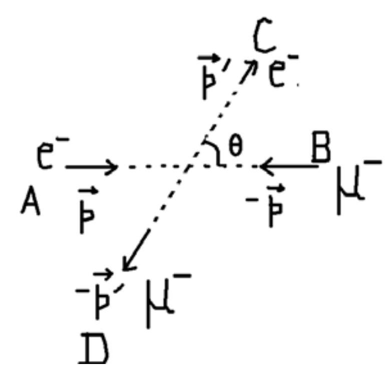
Here $q = p_D - p_B = p_A - p_C$.

In CM frame, for very high energy,

$p_A = (\surd s / 2,\ \mathbf{p})$, $p_C = (\surd s / 2,\ \mathbf{p}')$

$p_B = (\surd s / 2,\ -\mathbf{p})$, $p_D = (\surd s / 2,\ -\mathbf{p}')$,

$/\mathbf{p}/ = /\mathbf{p}'/ = p = \surd s / 2$.



The momenta of the particles are shown in the second figure. Therefore

$$q^2 = p_D^2 + p_B^2 - 2p_D.p_B = -\frac{s}{2}(1-\cos\theta)$$

and $(p_A + p_C).(p_B + p_D) = \frac{s}{2}(3+\cos\theta)$.

Substituting the values we get, $|M|^2 = (4\pi\alpha)^2 \left(\frac{3+\cos\theta}{1-\cos\theta} \right)^2$.The differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |M|^2$$

$$= \frac{\alpha^2}{4s} \left(\frac{3+\cos\theta}{1-\cos\theta} \right)^2.$$