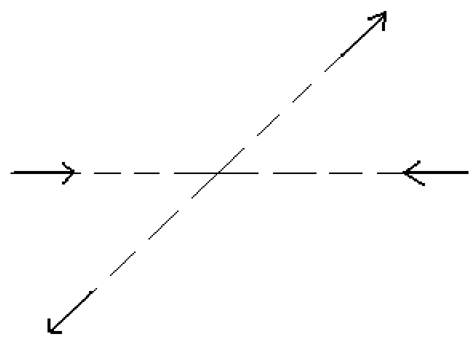
Problem 12

Consider $e^-(E_A, \mathbf{k}_A) + e^+(E_B, \mathbf{k}_B) \rightarrow e^-(E_C, \mathbf{k}_C) + e^+(E_D, \mathbf{k}_D)$ to be the s-channel process. Find s, t, u variables in the CM frame. Show that the process is physically allowed provided $s \geq 4m^2$, $t \leq 0$, $u \leq 0$ (where m = mass of electron).

➢ To work out the kinematics, consider the figure below:



In CM frame

$|\mathbf{k}_i| = |\mathbf{k}_f| = k$, where $\mathbf{k}_i$ = momentum of the incident electron and $\mathbf{k}_f$ = momentum of the scattered electron. Now

$s = (p_A + p_B)^2$

$= m^2 + m^2 + 2\, p_A \cdot p_B$

$= 2\,[\, m^2 + E^2 + k^2 \,]$

or $s = 4\,(k^2 + m^2)$.

$t = (p_A - p_C)^2 = 2\,(m^2 - p_A \cdot p_B) = 2\,[\, m^2 - (E^2 - \mathbf{k}_i \cdot \mathbf{k}_f)\,] = 2\,[\, m^2 - (E^2 - k^2 \cos\theta)]$

or $t = -2k^2(1 - \cos\theta)$.

$u = (p_A - p_D)^2 = 2\,(m^2 - p_A \cdot p_D) = 2\,[\, m^2 - (E^2 - k^2 \cos(\pi - \theta)]$

or $u = -2k^2(1 + \cos\theta)$.

Therefore, for the process to occur

$s \geq 4m^2$, $t \leq 0$, $u \leq 0$.

Comments:

(i) $t = 0$ ($u = 0$) corresponds to forward (backward) scattering.

(ii) For the s-channel process $A(e^-)\ B(e^+) \rightarrow C(e^-)\ D(e^+)$, consider the crossing (or interchange) of particles B and D, i.e., consider the process

$$A(e^-)\ \overline{D}(e^-) \rightarrow C(e^-)\ \overline{B}(e^-)$$

then s, t, u variables for the crossed channel may be found by using the replacement $p_B \leftrightarrow -p_D$. That is $s = (p_A + (-p_D))^2$, $t = (p_A - p_C)^2$, and $u = (p_A - (-p_B))^2$; or

$s = -2k^2 (1 + \cos\theta)$, $t = -2k^2 (1 - \cos\theta)$, $u = 4(k^2 + m^2)$.

For the process to occur,

$s \le 0$, $t \le 0$, and $u \ge 4m^2$. This crossed process is called a u channel process. For this process, u is the total center of mass energy.

The kinematics or physical regions of processes related by crossing can be shown on a two dimensional plot which maintains the symmetry of s, t, u. The three axis, s, t, u = 0 are drawn to form an equilateral triangle of height $\Sigma (m_i)^2$. From any point inside and also outside (pay proper attention to the sign of s, t, u) the triangle, the sum of perpendicular distances to the axes is equal to the height of the triangle.

The Mandelstam plot showing the physical regions for $e^- + e^+ \rightarrow e^- + e^+$ and the crossed reactions is shown below.

(see Fig.4.7, page 95, Halzen and Martin, Quarks and Leptons, John Wiley & Sons.)