Problem 1

Discuss Feynman – Stuckelberg interpretation of negative energy solutions of a free particle Klein-Gordon equation.

The free particle Klein Gordon equation is

$$(\Box^2 + m^2)\ \phi(x) = 0.$$

The probability current density $j^\mu = (\rho, \mathbf{j})$ for this equation is

$$j^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*).$$

For the plane wave solutions $\phi(x) = N e^{-ip.x}$,

$$j^\mu = 2 p^\mu |N|^2.$$

Use of $\phi(x) = N e^{-ip.x}$ in

$$(\partial^\mu \partial_\mu + m^2)\phi = 0$$

admits two solutions

$$E = \pm \sqrt{(|\mathbf{p}|^2 + m^2)}.$$

For $E < 0$, note that $\rho < 0$. The $\rho < 0$ does not have a physical meaning.

Pauli- Weiskopf (1934) suggestion

For electron-like spin 0 particle having electric charge –e, define probability charge current density (i.e., electromagnetic four-current density) as

$$j^\mu = i(-e)(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$$

and interpret ρ as a measure of electric charge density and **j** as a measure of electric current density.

For electron-like spin 0 particle with positive energy E, three momentum **p** and electric charge - e , the four-current density is

$$j^{\mu}(e^{-}_{s=0}, \text{energy} > 0) = -2e|N|^2(E, \vec{p}).$$

For positron-like spin 0 particle with positive energy E, three momentum **p** and electric charge + e,

$$j^{\mu}(e^{+}_{s=0}, \text{energy} > 0) = +2e|N|^2(E, \vec{p}).$$

For electron-like spin 0 particle with negative energy - E, three momentum - **p** and electric charge - e ,

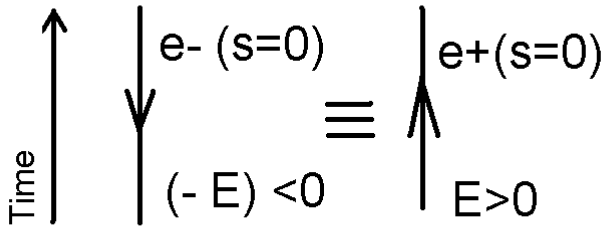
$$j^{\mu}(e^{-}_{s=0}, \text{energy} < 0) = -2e|N|^2(-E, -\vec{p})$$

$$= j^{\mu}(e^{+}_{s=0}, \text{energy} > 0).$$

The negative energy particle solutions are equivalent to positive energy anti-particle solutions. In addition note that

$$e^{-iEt} = e^{-i(-E)(-t)},$$

LHS term contains forward moving time while RHS contains backward moving time. The Feynman-Stuckelberg interpretation is that a negative energy particle moving backward in time is equivalent to a positive energy antiparticle moving forward in time. Pictorially



Absorption of a negative energy particle Emission of a positive energy antiparticle

(Try! Give Feynman-Stuckelberg interpretation for negative energy solutions of free particle Dirac equation.)