Detailed explanation of principles involved in this experiment

1. Measuring resistance – Meaning of Two probe, Four probe -methods:

Ohm's law: If physical conditions (like temperature, mechanical stress) remains unchanged, then potential difference across two ends of a conductor is proportional to current flowing through a conductor.

$$V \propto I \ ,$$

or

$$V = I R \ .$$

The constant of proportionality, *R,* is called resistance of the conductor.

Resistivity: At a constant temperature, the resistance, *R,* of a conductor is (i) proportional to its length and (ii) inversely proportional to its area of cross-section,

$$R \propto \frac{L}{A} \ , \qquad \text{or} \qquad R = \rho \frac{L}{A} \ .$$

The constant of proportionality, ρ , is called resistivity of material of the conductor.

Resistivity of a material is equal to the resistance offered by a wire of this material of unit length and unit cross-sectional area.

Unit of resistance is ohm (Ω), and unit of resistivity is ohm.meter ($\Omega \cdot m$)

The Ohm's law can also be expressed as $\vec{E} = \rho \vec{J}$ where $\vec{J}$ is the current density and $\vec{E}$ is the electric field set up in the resistance.

The scalar equation $E = \rho J$ is only valid for resistors and conductors which are homogeneous and isotropic, that is the conductivity (reciprocal of resistivity) is uniform throughout the conductor, carrying a uniform electric current per unit cross-sectional area, *J*, through a uniform internal electric field *E*, all in one dimension only.

Two probe method

For a long thin wire-like geometry of uniform cross-section or for a long parellelopiped shaped sample of uniform cross-section, the resistivity ρ can be measured by measuring voltage drop across the sample due to passage of known (constant) current through the sample, as shown in Fig.1. XY is the specimen whose resistivity is to be measured. The battery E supplies current (in through probe **1** and out through probe **2**). Let the current in the specimen is *I* (ampere). It is measured by the ammeter A. The potential difference between the two contacts (probe **1** and probe **2**) at the ends of the specimen is *V* (volt). It is measured by the voltmeter V. Let *l* is length of the specimen between the two probes and *A* its area of cross-section, then, the resistivity of the specimen is

$$\rho = \frac{V}{I} \frac{A}{l} \ .$$

Drawbacks of two probe method

(i) The major problem in such method is error due to contact resistance of measuring leads.

(ii) The above method cannot be used for for materials having random shapes.

(iii) For some type of materials soldering the test leads would be difficult.

(iii) In case of semiconductors, the heating of samples due to soldering results in injection of impurities into the materials thereby affecting the intrinsic electrical resistivity. Moreover, certain metallic contacts form Schottky barrier on semiconductors.

To overcome first two problems, a collinear equidistant four-probe method is used. This method provides the measurement of the resistivity of the specimen having wide variety of shapes but with uniform cross-section. The soldering contacts are replaced by pressure contacts to eliminate the last two problems discussed above.

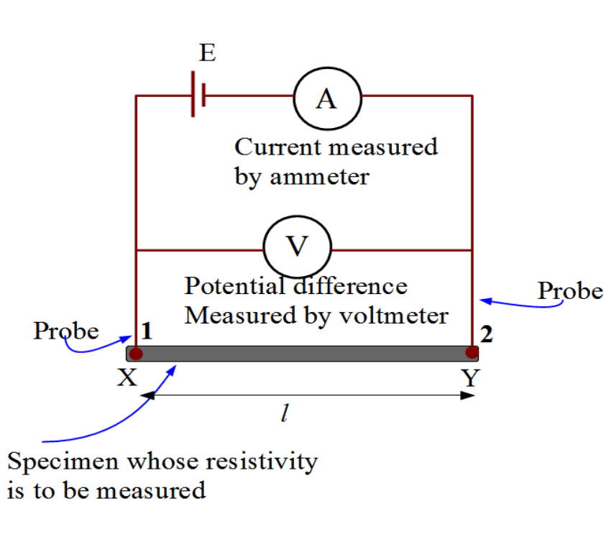

Fig.1 Two probe method of measuring resistivity of a specimen.

Four probe method

The 4-point probe set up (Fig.2) consists of four equally spaced tungsten metal tips with finite radius. Each tip is supported by springs on the end to minimize sample damage during probing. The four metal tips are part of an auto-mechanical stage which travels up and down during measurements. A high impedance current source is used to supply current through the outer two probes, a voltmeter measures the voltage across the inner two probes to determine the sample resistivity. Typical probe spacings ~ 1 mm

These inner probes draw no current because of the high input impedance voltmeter in the circuit. Thus unwanted voltage drop (*I R* drop) at point B and point C caused by contact resistance between probes and the sample is eliminated from the potential measurements. Since these contact resistances are very sensitive to pressure and to surface condition (such as oxidation of either surface), error with the conventional two-electrode technique (in which potential-measuring contact passes a current) can be quite large.

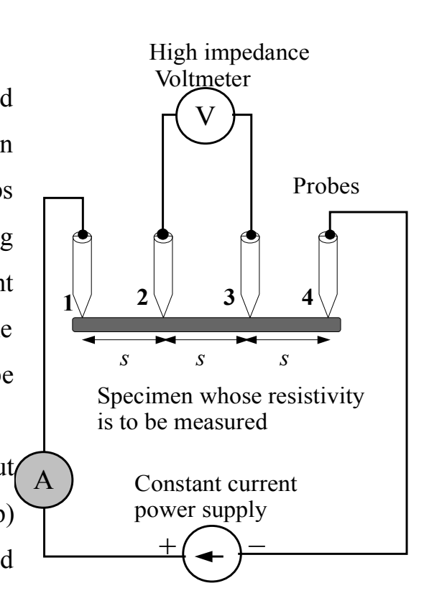

Fig.2 Four probe method of measuring resistivity of a specimen.

The electric current carried through the two outer probes, sets up an electric field in the sample. In Fig.3, the electric field lines are drawn solid and the equipotential lines are drawn broken. The two inner probes measure the potential difference between point B and C.

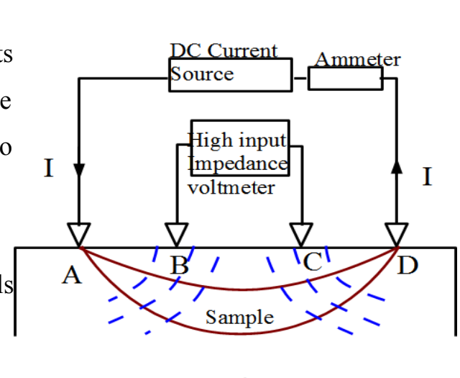

Fig.3

2. Resistivity of Germanium (semiconductor) crystals or slices:

In order to use this four probe method in germanium crystals or slices it is necessary to assume that:

1. The resistivity of the material is uniform in the area of measurement.
2. If there is minority carrier injection into the semiconductor by the current-carrying electrodes most of the carriers recombine near the electrodes so that their effect on the conductivity is negligible. (This means that the measurements should be made on surfaces which have a high recombination rate, such as mechanically lapped surfaces.)
3. The surface on which the probes rest is flat with no surface leakage.
4. The four probes used for resistivity measurements contact the surface at points that lie in a straight line.
5. The diameter of the contact between the metallic probes and the semiconductor should be small compared to the distance between probes.
6. The boundary between the current-carrying electrodes and the bulk material is hemispherical and small in diameter.
7. The surfaces of the germanium crystal may be either conducting or nonconducting. (a) A conducting boundary is one on which a material of much lower resistivity than germanium (such as copper) has been plated. (b) A nonconducting boundary is produced when the surface of the crystal is in contact with an insulator.

The derivation of equations given below are involved. Details are given in the original work of L B Valdes. Reference: L. B. Valdes, Resistivity Measurements on Germanium for Transistors, Proceedings of the I R E, pp.420 – 427, February 1954. Only the final results for each of the cases are presented here. For each case it is assumed that the probes are equally spaced (spacing = *s*).

Case I. Resistivity Measurements on a Large Sample

We assume that the metal tip is infinitesimal and sample are semi infinite in lateral dimensions. For bulk samples where the sample thickness, $w >> s$, the probe spacing, we assume a spherical protrusion of current emanating from the outer probe tips.

The resistivity is computed to be

$$\rho_0 = \left(\frac{V}{I} \right) 2 \pi s \ ,$$

where

V = floating potential difference between the inner probes, unit: volt

I =current through the outer pair of probes, unit: ampere

s = spacing between point probes, unit: meter

ρ_0 = resistivity, unit: ohm meter

Case II. Resistivity Measurements on a Thin Slice-Conducting Bottom Surface

Fig.4 shows the resistivity probes on a die of material. If the side boundaries are adequately far from the probes the die may be considered to be identical to a slice. For this case of a slice of thickness *w* and with a conducting bottom surface the resistivity is computed by means of the divisor $G_6(w/s)$ of Fig.5 as:

$$\rho = \frac{\rho_0}{G_6(w/s)} \ .$$

This method is not recommended for *w/s* very small.

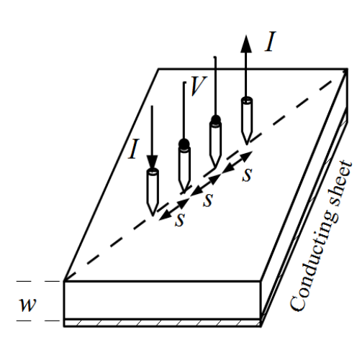

Fig.4 Resistivity measurement on a thin slice

Case III. Resistivity Measurements on a Thin Slice-Nonconducting Bottom Surface

For the case of a nonconducting bottom on a slice like that of Case II, the resistivity is computed from

$$\rho = \frac{\rho_0}{G_7(w/s)} \ .$$

The function $G_7(w/s)$ is shown in Fig.6.

The values of the functions $G_6(w/s)$ and $G_7(w/s)$ are tabulated in Table 1.

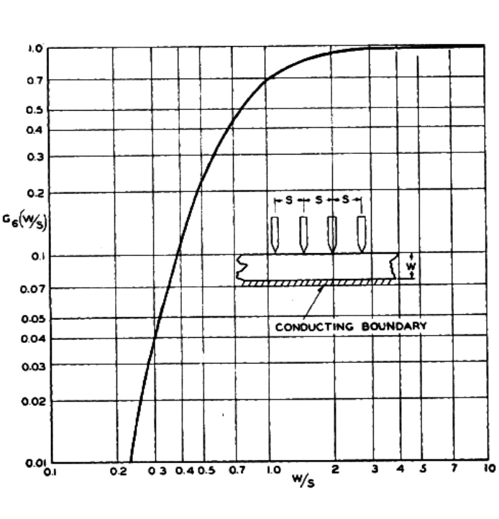

Fig.5 Correction divisor for probes on thin slice with conducting bottom.

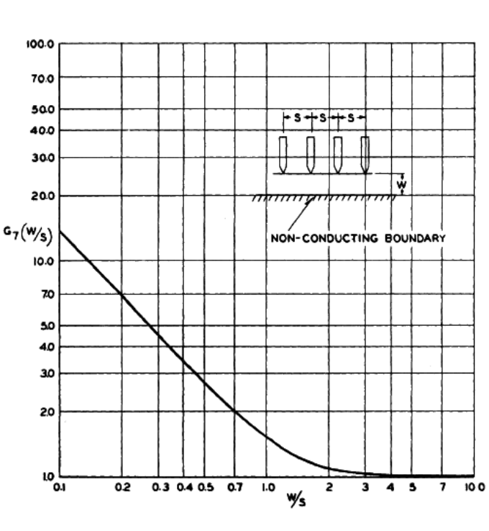

Fig.6 Correction divisor for probes on a thin slice with non-conducting bottom surface.

Table 1: Values of functions $G_6(w/s)$ and $G_7(w/s)$.

For smaller values of *w/s* the function $G_7(w/s)$ approaches the case for an infinitely thin slice, that is,

$$G_7(w/s) = \frac{2s}{w} \ln 2 \ , \text{and}$$

$$\rho = \frac{\pi w}{\ln 2} \left(\frac{V}{I} \right) = 4.5324 \times w \times \left(\frac{V}{I} \right) \ .$$

3. Temperature dependence of resistivity of a semiconductor:

Intrinsic semi-conduction

The process in which thermally or optically excited electrons contribute to the conduction is called intrinsic semi-conduction.

In the absence of photonic excitation, intrinsic semi-conduction takes place at temperatures above 0 K as sufficient thermal agitation is required to transfer electrons from the valence band to the conduction band.

Conductivity for intrinsic semi-conduction

The total electrical conductivity is the sum of the conductivities of the valence and conduction band carriers, which are holes and electrons, respectively.

It can be expressed as

$$\sigma = e \left(n_e \mu_e + n_h \mu_h \right) \qquad (3.1)$$

where n_e, μ_e are the electron's concentration and mobility, and n_h, μ_h are the hole's concentration and mobility, respectively.

The mobility is a quantity that directly relates the drift velocity v_d of charge carriers to the applied electric field *E* across the material, i.e.,

$$\mu = \frac{v_d}{E} \ . \qquad (3.2)$$

In the intrinsic region the number of electrons is equal to the number of holes, $n_e = n_h \equiv n_i$, so Equation (3.1) implies that,

$$\sigma = n_i \, e \left(\mu_e + \mu_h \right) \ . \qquad (3.3)$$

Temperature dependence of conductivity

The electron density (electrons/volume) in the conduction band is obtained by integrating (density of states × probability of occupancy of states) from the bottom to top of the conduction band. The detailed calculations reveal that

$$n_i = N \, T^{3/2} \exp\left(-\frac{E_g}{2kT} \right) \ , \qquad (3.4)$$

where *N* is some constant. Substituting (3.4) in (3.3) gives

$$\sigma = N \, e \left(\mu_e + \mu_h \right) T^{3/2} \exp\left(-\frac{E_g}{2kT} \right) \ . \qquad (3.5)$$

Eq.(3.5) shows that the electrical conductivity of intrinsic semiconductors depends on temperature and decreases exponentially with decreasing temperature.

Temperature dependence of mobility

Drift mobility determines the average drift velocity in the presence of an applied external field. It also depends on the temperature. In the intrinsic semi-conduction region, the charge carrier, suppose an electron, suffers collisions from the scattering ions (such as impurity ion). These scattering events depend on how strongly the ions vibrate, the amplitude depends on the temperature *T*. The mean free time between scattering events, is given by,

$$\tau = \frac{1}{S \, v_{th} \, N_{sc}} \ , \qquad (3.6)$$

where *S* is the cross-sectional area of the scatterer shown in Fig.7, v_{th} is the mean speed of the electrons, called the thermal velocity and N_{sc} is the number of scatterers per unit volume.

Now both the scatterer amplitude *a* and the thermal velocity of the electron v_{th} are temperature dependent. The scatterer amplitude increases with temperature as $a^2 \propto T$. Now an electron in the conduction band has only kinetic energy and the mean kinetic energy per electron in the conduction band is $(3/2)kT$. Applying kinetic molecular theory to the gas of electrons in the conduction band, we obtain,

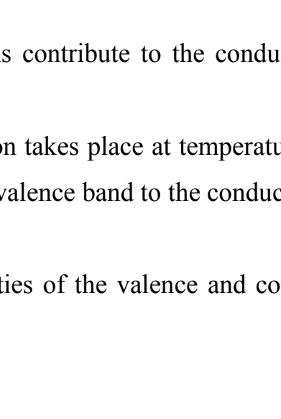

Fig.7 Scattering cross sections due to lattice vibrations

$$\frac{1}{2} m_e^* v_{th}^2 = \frac{3}{2} kT \ , \qquad (3.7)$$

implying $v_{th} \propto T^{1/2}$. Using the above derived temperature dependencies of *a* and v_{th} , the scatterer mean time τ (Eq.(3.6)) due to lattice vibrations becomes,

$$\tau \propto T^{-3/2} \ . \qquad (3.8)$$

According to Drude model, the drift mobility is

$$\mu = \frac{e \tau}{m_e^*} \ . \qquad (3.9)$$

The use of Eq.(3.8) in Eq.(3.9) gives the following temperature dependence of the mobility in the intrinsic semi-conduction region:

$$\mu \propto T^{-3/2} \ . \qquad (3.10)$$

Temperature dependence of Resistivity

The resistivity is reciprocal of conductivity. For intrinsic semiconductor, it is (from Eq.(3.5))

$$\rho = \frac{1}{N \, e \left(\mu_e + \mu_h \right) T^{3/2}} \exp\left(\frac{E_g}{2kT} \right) \ . \qquad (3.11)$$

Since the mobility dependence on temperature according to Eq.(3.10), we expect

$$\left(\mu_e + \mu_h \right) T^{3/2} \approx constant \ .$$

Therefore, Eq.(3.11) can be written as

$$\rho = A \exp\left(\frac{E_g}{2kT} \right) \ . \qquad (3.12)$$

where *A* is some constant. Taking logarithm, we get

$$\log_e \rho = \log_e A + \frac{E_g}{2kT} \ ,$$

or

$$\log_{10} \rho = C + \frac{1}{2.3026} \frac{E_g}{2kT} \qquad (3.13)$$

where *C* is a constant. For convenience Eq.(3.13) is rewritten as

$$\log_{10} \rho = C + \frac{1}{2.3026 \times 10^3} \frac{E_g}{2k} \times \frac{10^3}{T} \ . \qquad (3.14)$$

Thus a graph between log of resistivity, $\log_{10} \rho$, and reciprocal of the temperature, $\frac{10^3}{T}$, should be a straight line (see Fig.8).

Band gap energy

The slope of the straight line graph between log of resistivity, $\log_{10} \rho$, and reciprocal of the temperature, $\frac{10^3}{T}$, is (see Fig.8)

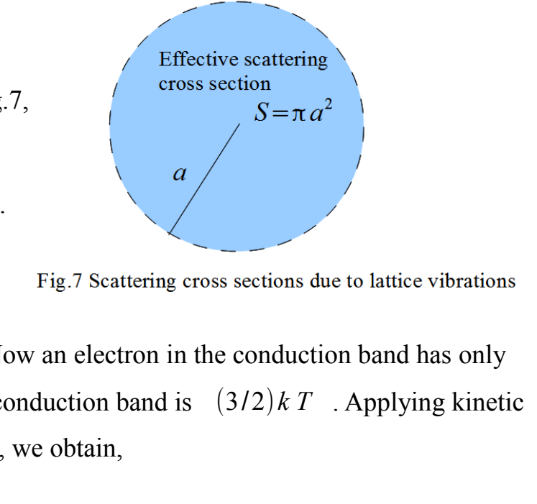

Fig.8 Graph between $\log_{10} \rho$ and $10^3/T$.

$$slope = \frac{(AC)}{(BC)} = \frac{1}{2.3026 \times 10^3} \frac{E_g}{2k} \ .$$

Therefore,

$$E_g = 2.3026 \times 10^3 \times 2k \times (slope) \ .$$

Use $k = 8.617 \times 10^{-5}$ eV K^{-1} to get E_g in eV unit.

4. Description of the apparatus:

The experimental set up (Fig.9) consists of (i) probe arrangement, (ii) sample , (iii) oven, thermometer (0 – 200 °C), (iv) constant current generator, (v) oven power supply, (vi) digital panel meter (for measuring voltage & current).

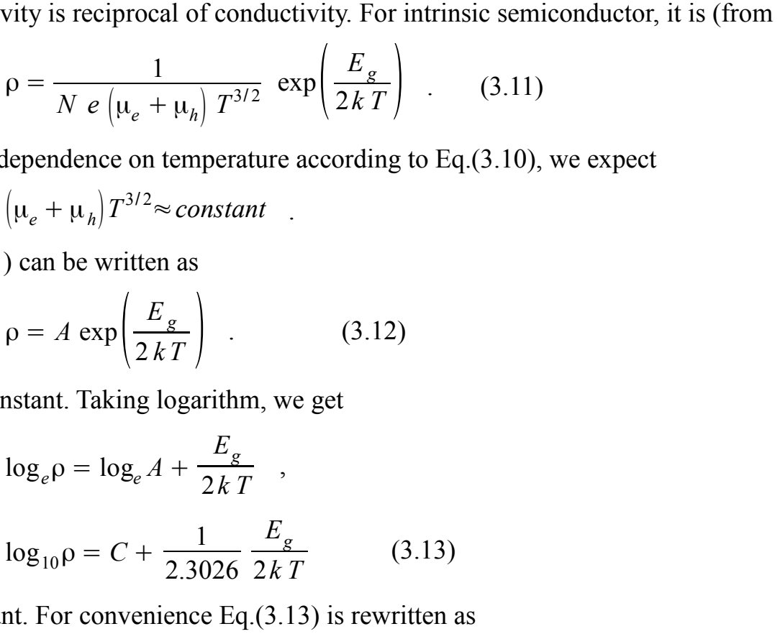

Fig.9 Schematic diagram of a Four Probe Apparatus

The four-probe assembly consists of four spring loaded probes arranged in a line with equal spacing between adjacent probes. These probes rest on a metal plate on which thin slices of samples (whose resistivity is to be determined) can be mounted by insulating their bottom surface using a mica sheet. Different colored leads are provided for carrying current and for voltages measurements. The sample, usually, is brittle, hence do not attempt to mount the sample yourself. This assembly is mounted in a lid of an oven, so that the four probes and the sample can be kept inside the oven and sample can be heated up to a temperature of 200 °C. The temperature inside the oven can be measured by inserting a thermometer through a hole in the lid.

The sample is, normally, germanium crystal in the form of a chip. It is brittle and costly. Therefore, the setup should be handled only after fully understanding the management of probe settings. Contacts of the probes with the crystal should be done very carefully using gentle up /down motion of the screw provided for the purpose.

There exists provision for varying the temperature of the oven (up to a maximum of 200 °C). Suitable voltage for the oven is obtained through a step down transformer with a provision for low & high rates of heating.

5. Methodology adopted to measure ρ :

The basic methodology is to use four-probe method for measurement of current and voltage.

(I) The current and voltage are measured across a germanium test-piece at different values of temperature.

(II) From the measurements, the resistivity, ρ , is calculated by using the following relation corresponding to each temperature:

$$\rho = \frac{\rho_0}{G_7(w/s)} = \left(\frac{V}{I} \right) \frac{2 \pi s}{G_7(w/s)} \ .$$

(III) The variation of semiconductor resistivity with temperature is inferred by plotting a graph between log of resistivity, $\log_{10} \rho$, and reciprocal of the temperature, $10^3/T$. A linear plot is obtained.

(III) From the linear plot, slope is determined. Using it, energy gap of germanium is calculated.

$$E_g = 2.3026 \times 10^3 \times 2k \times (slope) \ .$$

Experiment A - 4

Object: Study of the temperature dependence of resistivity of a semiconductor (Four probe method).

Apparatus: Four probe apparatus (spring loaded four probes, germanium crystal in the form of a chip, oven for variation of temperature (to about 150 °C), thermometer, constant current power supply, oven power supply, high impedance voltmeter, milli-ammeter to measure, respectively, voltage and current through 4-point probes).

Theory:

Resistivity of semiconductor

The resistivity of germanium is measured with the help of four probes (Fig.I). The outer probes are used for passing a constant current through the germanium sample. The electric current carried through the two outer probes, sets up an electric field in the sample. In Fig.II, the electric field lines are drawn solid and the equipotential lines are drawn broken. The two inner probes measure the potential difference between point B and C using a high impedance voltmeter.

For bulk samples where the sample thickness, $w >> s$, the probe spacing, the resistivity is calculated using the relation:

$$\rho_0 = \left(\frac{V}{I} \right) 2 \pi s \ , \qquad (1)$$

where

V = floating potential difference between the inner probes, unit: volt

I =current through the outer pair of probes, unit: ampere

s = spacing between point probes, unit: meter

ρ_0 = resistivity, unit: ohm meter

Fig.III shows the resistivity probes on a die of material. If the side boundaries are adequately far from the probes the die may be considered to be identical to a slice. For this case of a slice of thickness *w* and with a non conducting bottom surface the resistivity is computed by means of the divisor $G_7(w/s)$ as:

$$\rho = \frac{\rho_0}{G_7(w/s)} = \left(\frac{V}{I} \right) \frac{2 \pi s}{G_7(w/s)} \ . \qquad (2)$$

The values and graph for $G_7(w/s)$ are given in the lab manual. For $(w/s) \leq 0.5$, we may use the following value (obtained for the case of infinitely thin slice):

$$G_7(w/s) = \frac{2s}{w} \ln 2 = \frac{1.3863}{(w/s)} \ . \qquad (3)$$

The temperature dependence of resistivity of a semiconductor

The total electrical conductivity of a semiconductor sample is the sum of the conductivities of the valence and conduction band carriers, which are holes and electrons, respectively.

$$\sigma = e \left(n_e \mu_e + n_h \mu_h \right) \qquad (4)$$

where

n_e, μ_e are the electron's concentration and mobility, and

n_h, μ_h are the hole's concentration and mobility,

μ_e, μ_h Are the electron and the hole mobilities.

In the intrinsic region the number of electrons is equal to the number of holes, $n_e = n_h \equiv n_i$, so the conductivity becomes

$$\sigma = n_i \, e \left(\mu_e + \mu_h \right) \ . \qquad (5)$$

The detailed calculations reveal that the electron density (number/volume) depends on temperature as follows:

$$n_i = N \, T^{3/2} \exp\left(-\frac{E_g}{2kT} \right) \ , \qquad (6)$$

where *N* is some constant. The temperature dependence of the mobility in the intrinsic semi-conduction region is of the form:

$$\mu \propto T^{-3/2} \ . \qquad (7)$$

Therefore

$$\left(\mu_e + \mu_h \right) T^{3/2} \approx constant$$

Use of this fact gives

$$\sigma = constant \times \exp\left(-\frac{E_g}{2kT} \right) \ . \qquad (8)$$

The resistivity is reciprocal of conductivity. Therefore, for intrinsic semiconductor, it is (from Eq.(8))

$$\rho = A \exp\left(\frac{E_g}{2kT} \right) \ . \qquad (9)$$

where *A* is some constant. The resistivity of a semiconductor rises exponentially on decreasing the temperature. Taking logarithm, we get

$$\log_{10} \rho = C + \frac{1}{2.3026} \frac{E_g}{2kT} \qquad (10)$$

where $C = \log_{10} A$ is another constant. For convenience Eq.(10) is rewritten as

$$\log_{10} \rho = C + \frac{1}{2.3026 \times 10^3} \frac{E_g}{2k} \times \frac{10^3}{T} \ . \qquad (11)$$

Thus a graph between log of resistivity, $\log_{10} \rho$, and reciprocal of the temperature, $\frac{10^3}{T}$, should be a straight line (Fig.IV).

Band gap energy

The slope of the straight line graph between log of resistivity, $\log_{10} \rho$, and reciprocal of the temperature, $\frac{10^3}{T}$, is (see Fig.IV)

$$slope = \frac{(AC)}{(BC)} = \frac{1}{2.3026 \times 10^3} \frac{E_g}{2k} \ .$$

Therefore,

$$E_g = 2.3026 \times 10^3 \times 2k \times (slope) \ .$$

We use $k = 8.617 \times 10^{-5}$ eV K^{-1} to get E_g in eV unit.

Figures:

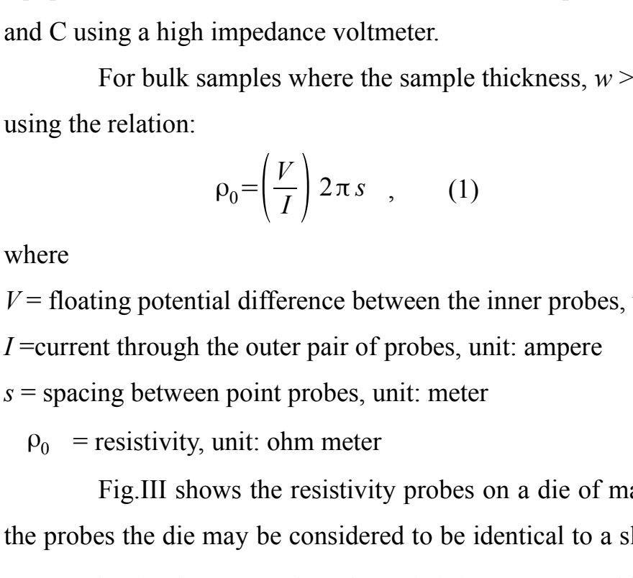

Fig.I Schematic diagram of a Four Probe Apparatus

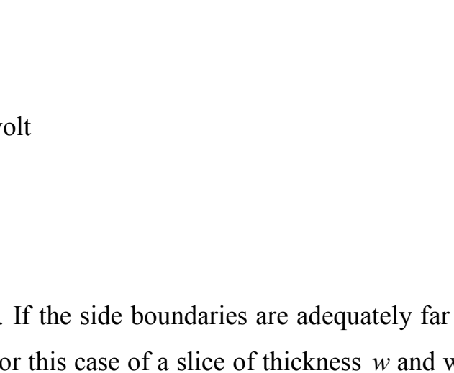

Fig.II

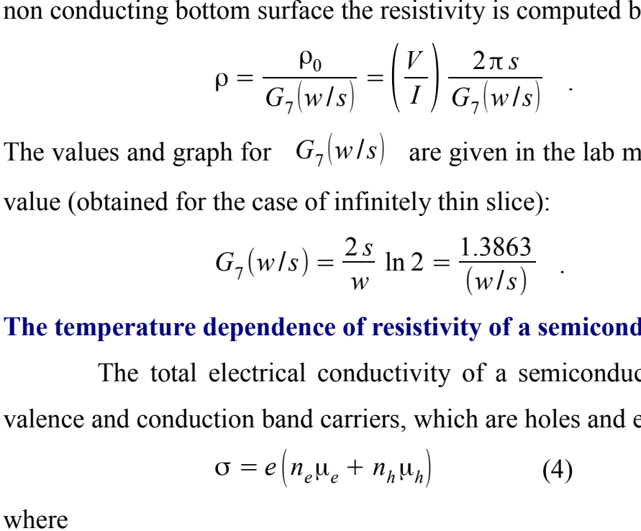

Fig.III Resistivity measurement on a thin slice

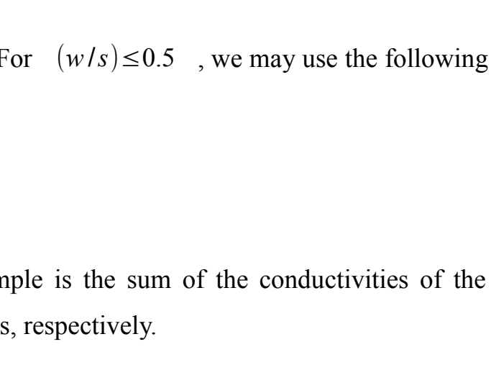

Fig.IV Graph between $\log_{10} \rho$ and $10^3/T$.

Method:

(1) The settings of 4-point probes on the semiconductor chip is a delicate process. So first understand well the working of the apparatus. The semiconductor chip and probe set is costly.

(2) Note the values of probe spacing (*s*) and the thickness (*w*) of the semiconductor chip. Note the type of semiconductor (germanium or something else).

(3) Make the circuit as shown in Fig.I & III. Put the sample in the oven (normally already placed by lab instructor) at room temperature.

(4) Carefully, with the help of screw and delicate up/down movements, touch (kelvin connection) the probes to the semiconductor chip.

(5) Pass a milliampere range current (say 2 mA) in the sample using constant current power supply.

(6) The reading of the current through the sample is measured using milliammeter provided for this purpose. The voltage is measured by a high impedance millivoltmeter connected to the inner probes. The readings can be taken alternately on digital meter provided for this purpose.

(7) Note temperature of sample (oven) using thermometer inserted in the oven for this purpose.

(8) The oven temperature is increased a little, and its temperature noted after reaching steady state. Again the constant current reading (advised to be kept the same) and the corresponding voltage readings are taken.

(9) Repeat the procedure for different temperatures. Note the data in the observation table.

(10) For each temperature, calculate the resistivity by using the relation

$$\rho = \frac{\rho_0}{G_7(w/s)} = \left(\frac{V}{I} \right) \frac{2 \pi s}{G_7(w/s)} \ .$$

(11) Compute $\log_{10} \rho$ and $10^3/T$ and write it in the observation table.

(12) Plot a graph between $\log_{10} \rho$ and $10^3/T$. It is a straight line. Find its slope.

(13) Calculate the band gap using formula

$$E_g = 2.3026 \times 10^3 \times 2k \times (slope) \ .$$

Use use $k = 8.617 \times 10^{-5}$ eV K^{-1} to get E_g in eV unit. Note that up to four-significant digits, $k = 1.3806 \times 10^{-23}$ J K^{-1} , and $1 \, \text{eV} = 1.602 \times 10^{-19}$ J .

Observations:

1. Semiconductor chip material = Germanium (verify from your lab manual)
2. Spacing (distance) between the probes, *s* = …..........mm = …..........m.
3. Thickness of the sample, *w* = …..........mm = …..........m.

Table 1: Voltage across the inner probes for a constant current at different sample temperatures

(a) Constant current passed through the sample = …........mA

S.NO.	Temperature T (K)	Voltage across inner probes V (mV)	$\frac{10^3}{T}$ (K^{-1}) (calculated)	Resistivity ρ (Ω m) (calculated)	$\log_{10} \rho$ (calculated)
1					
2					
3					
...					
...					
...					

(If instructed, make one more table with a different value of constant current in the sample.)

Calculations:

1. For the given sample $\frac{w}{s}$ = ...
2. The correction factor $G_7(w/s)$ =........
3. Calculation of $\frac{10^3}{T}$ (K^{-1}) , ρ (Ω m) and $\log_{10} \rho$

   (I) For S.No. 1 of Table 1

   (i) Temperature *T* = …......K. Therefore

   $$\frac{10^3}{T} = ... (K^{-1}) \ .$$

   (ii) Resistivity is

   $$\rho = \left(\frac{V}{I} \right) \frac{2 \pi s}{G_7(w/s)} = ... \Omega \, m$$

   $$\log_{10} \rho = ...$$

   (II) Repeat calculations for other S.Nos.

4. Slope of the graph between $\frac{10^3}{T}$ (K^{-1}) and $\log_{10} \rho$) is

   $$slope = \frac{AC}{BC} = \frac{.........}{........} = ...$$

5. Energy band gap

   $$E_g = 2.3026 \times 10^3 \times 2k \times (slope)$$

   $$= 2.3026 \times 10^3 \times 2 \times 8.617 \times 10^{-5} \times (slope) = ... \text{ eV}$$

Result:

1. The temperature dependence of the resistivity of semiconductor (germanium) chip is as shown in the graph. The resistivity increases exponentially with the increase in 1/*T*. That is as at low temperatures resistivity is more and at high temperatures the resistivity is less.
2. The energy band gap for the given semiconductor (germanium) is = …......eV.

Precautions:

1. The surface of the semiconductor should be flat
2. All the four probes should be collinear.
3. The adjustment of 4-point probes should be done gently, as the semiconductor chip is brittle.
4. The voltage should be measured using inner probes only using a high impedance millivoltmeter.
5. Temperature of the oven should not exceed the limits set by manufacturer of the probes and chip set.